Precalculus
Lesson 10.4: The Hyperbola
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A **hyperbola** is the collection (locus) of all points In the plane, the difference of whose distances from two fixed points, called the foci, is a constant.

Equation of a Hyperbola Centered about the origin with Transverse Axis along the x-axis

who is first? who is positive? } x

Transverse axis variable is first

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

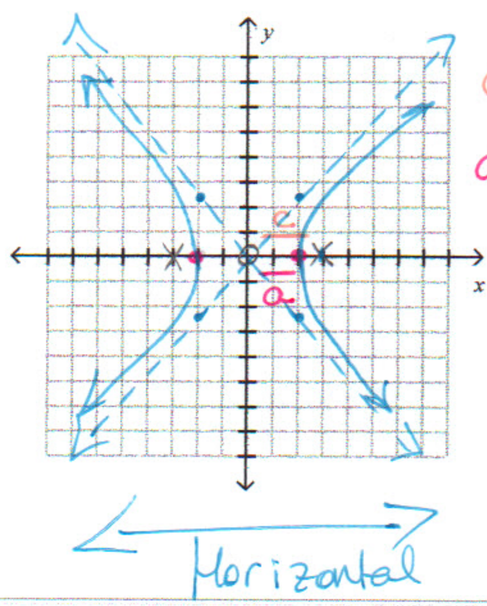
"a" is always in the 1st term position denominator doesnot matter its size.

$$b^2 = c^2 - a^2$$

center at $(0,0)$; foci at $(\pm c, 0)$; and vertices at $(\pm a, 0)$

two oblique asymptotes: $y = \pm\frac{b}{a}x$ ← slope $\frac{rise}{run} = \frac{y\ denom}{x\ denom}$

Find an equation of the hyperbola with center at the origin, one focus at $(3,0)$ and one vertex at $(-2,0)$. Graph



c = origin to focus = 3 $c^2 = 9$

a = origin to vertex = 2 $a^2 = 4$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{4} - \frac{y^2}{5} = 1$
run rise

$b^2 = c^2 - a^2$
$b^2 = 9 - 4$
$b^2 = 5$
$b = \pm\sqrt{5}$

Asymptote slope

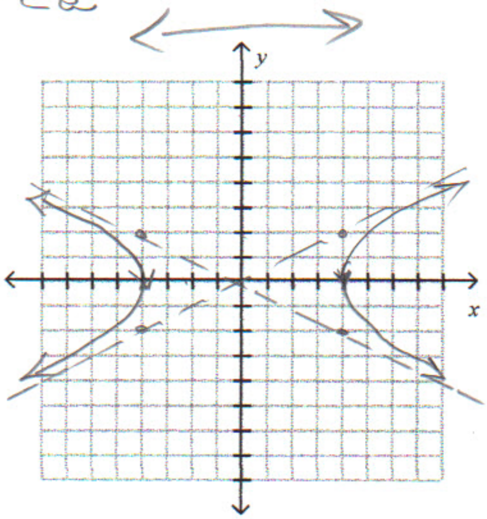
$\frac{rise}{run} = \frac{b}{a} = \frac{\pm\sqrt{5}}{2}$

Analyze the equation; find the center, transverse axis, vertices, and foci. Graph.

$$\frac{x^2}{16} - \frac{y^2}{4} = 1$$

$a = 4 \quad b = 2$

$\uparrow a$



$b^2 = c^2 - a^2$

$4 = c^2 - 16$

$20 = c^2$

$\pm 2\sqrt{5} = c$

Center $(0, 0)$

transverse axis $= x$

vertices $(\pm 4, 0)$

foci $(\pm 2\sqrt{5}, 0)$

Asymptote Slope $= \pm \frac{2}{4} = \pm \frac{1}{2}$

Equation of a Hyperbola; Center at (0, 0); Transverse Axis along the y-axis

first = positive variable

transverse axis is y

"a" sign will vary

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$b^2 = c^2 - a^2$$

center at $(0, 0)$; foci at $(0, \pm c)$; and vertices **at $(0, \pm a)$**

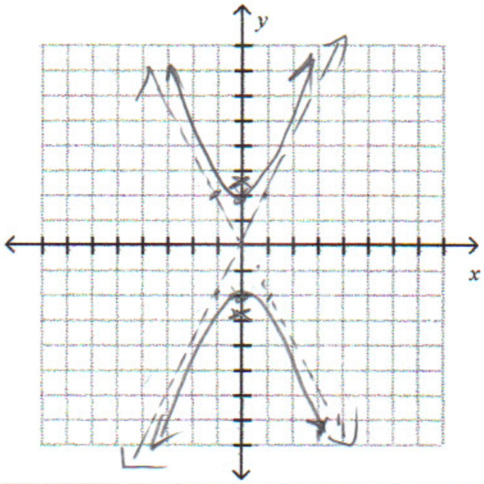
two oblique asymptotes: $y = \pm \frac{a}{b} x$

slope $= \frac{y}{x}$

Analyze the equation, find the center, transverse axis, vertices, and foci and graph:

$$\frac{y^2}{4} - \frac{4x^2}{4} = \frac{4}{4} \rightarrow \frac{y^2}{4} - \frac{x^2}{1} = 1$$

a^2 — 4



$b^2 = c^2 - a^2$

$1 = c^2 - 4$

$5 = c^2$

$\pm\sqrt{5} = c$

Asymptote Slope $= \pm\frac{a}{b} = \pm\frac{2}{1}$

– Center $(0,0)$

– transverse axis $= y$

– vertices (0 ± 2)

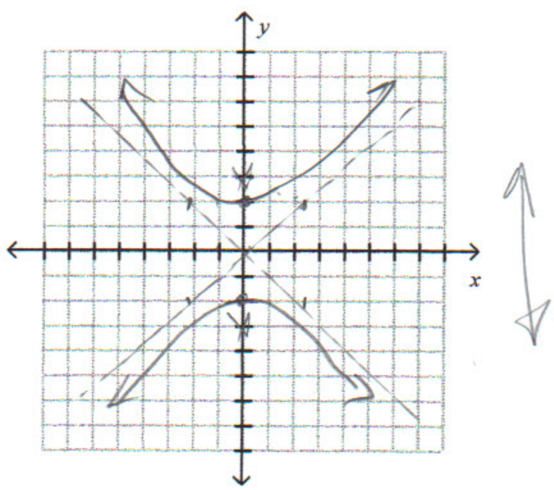
– foci $(0 \pm \sqrt{5})$

Find an equation of the hyperbola having one vertex at $(0,2)$ and foci at $(0,-3)$ *and* $(0,3)$. Graph.

$a = 2$ $c = 3$

$b^2 = 9 - 4$

$b^2 = 5$



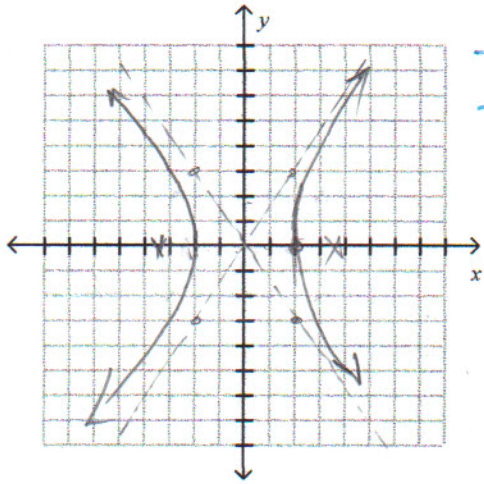
$$\frac{y^2}{4} - \frac{x^2}{5} = 1$$

Asymptote $= \frac{2}{\sqrt{5}} \pm \frac{2}{2.2}$

Analyze the equation, find the center, transverse axis , vertices, foci, and asymptotes and graph:

$$9x^2 - 4y^2 = 36$$

$$\frac{9x^2}{36} - \frac{4y^2}{36} = \frac{36}{36} \rightarrow \frac{x^2}{4} - \frac{y^2}{9} = 1$$



- Center $(0,0)$
- transverse axis $\Rightarrow 4$
- vertices $(\pm 2, 0)$
- foci $(\pm\sqrt{13}, 0)$
- Asymptote slope $\pm\frac{3}{2}$

$a = 2$

$b = 3$

$9 = c^2 - 4$

$13 = c^2$

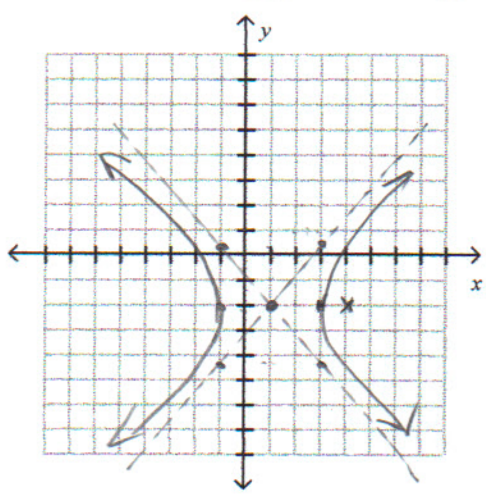
$\pm\sqrt{13} = c$

Hyperbolas at a center of (h, k)

Opens	**Opens left and right Transverse axis x-axis**	**Opens up and down Transverse axis y-axis**
Form:	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Center:	(h,k)	(h,k)
Vertices	$(h+a,k) and\ (h-a,k)$	$(h,k+a) and\ (h,k-a)$
Slope of Asymptotes	$\pm\frac{b}{a}$	$\pm\frac{a}{b}$
Equation of Asymptotes	$y-k=\pm\frac{b}{a}(x-h)$	$y-k=\pm\frac{a}{b}(x-h)$
Foci $a^2+b^2=c^2$	$(h+c,k),(h-c,k))$	$(h,k+c),(h,k-c)$

Find an equation for the hyperbola with center at $(1,-2)$, one focus at $(4,-2)$, and one vertex at $(3,-2)$. Graph the equation by hand.

$\xleftrightarrow{\text{transverse}}$ axis = x



Center to focus

1 to 4 = 3 = c

Center to vertex –

1 to 3 = 2 = a

$b^2 = 9 - 4$

$b^2 = 5$

$$\frac{(x-1)^2}{4}-\frac{(y+2)^2}{5}=1$$

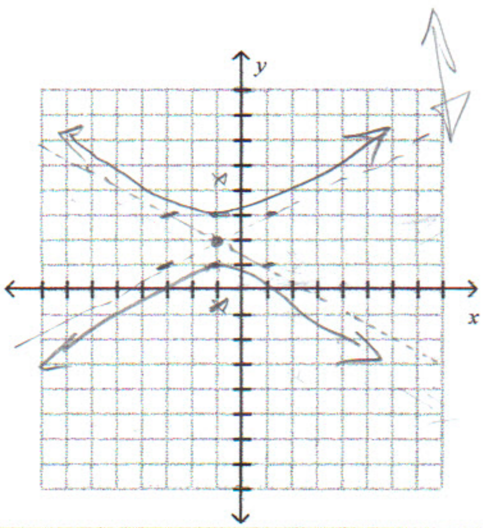
asymptotes = $\pm\frac{\sqrt{5}}{2} = \pm\frac{2.2}{2}$

Analyze the equation, find the center, transverse axis , vertices, foci, and asymptotes and graph:
$-x^2+4y^2-2x-16y+11=0$

$4y^2-16y+16-x^2-2x-1=-11+16-1$

$4(y^2-4y+4)-(x^2+2x+1)=4$

$\frac{4(y-2)^2}{4}-\frac{(x+1)^2}{4}=\frac{4}{4} \Rightarrow \frac{(y-2)^2}{1}-\frac{(x+1)^2}{4}=1$



- center $(-1,2)$
- transverse axis $\rightarrow y$
- vertices $\rightarrow a=1$

  $(-1,3)$ $(-1,1)$
- foci $(-1,2+\sqrt{5})$

  $(-1,2-\sqrt{5})$

$4=c^2-1$

$5=c^2$, $\pm\sqrt{5}=c$

Asymptotes slope $=\pm\frac{1}{2}$