Precalculus
Lesson 6.5: Graphs of the Tangent, Cotangent Cosecant, and Secant Functions
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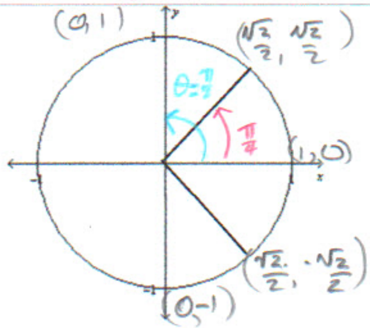
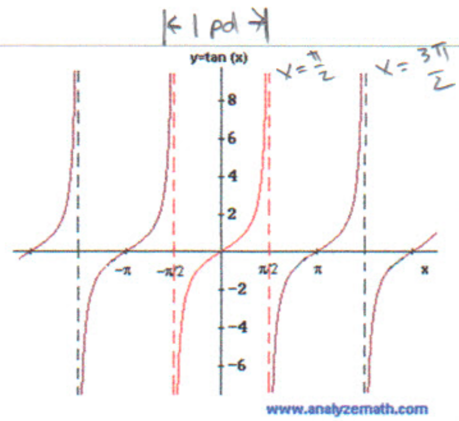
Tangent function facts:

- The tangent function has a period of π.
- $\tan x = \frac{\sin x}{\cos x}$, $\therefore$ *when* $\sin x = 0$, $\tan x = 0$
  and when $\cos x = 0$, $\tan x$ *is undefined*
- $\cos x = 0$ *at* $x = \frac{\pi}{2}$ *and* $-\frac{\pi}{2}$ so at these values tangent is undefined
- Tangent graph will have asymptotes at values of x where the function is undefined: $x = \frac{\pi}{2}$ *and* $-\frac{\pi}{2}$.



$\tan \frac{\pi}{2} = \frac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}} = \frac{1}{0} = u$

$\tan \frac{\pi}{4} = \frac{\sin \frac{\pi}{4}}{\cos \frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$



Period: π $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

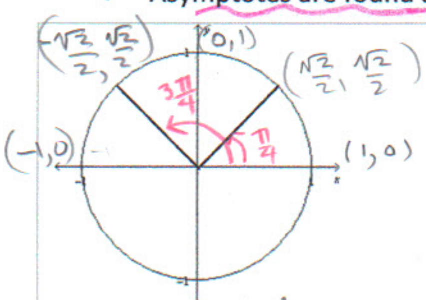
input x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y = \tan x$	undef	-1	0	1	undef

Domain: all real numbers except odd mult. of $\frac{\pi}{2}$

Range: $(-\infty, \infty)$

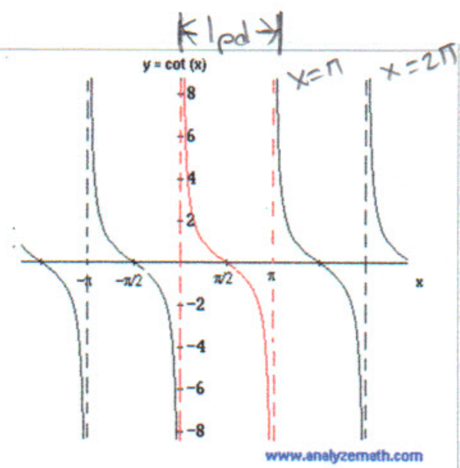
Cotangent function facts:

- The cotangent function also has a period of π.
- $\cot x = 0$ *when* $\cos x = 0$ *and* $\cot = undefined$ *when* $\sin x = 0$
- $\sin x = 0$ *at* $x = \pi$ *and* $-$ $\therefore$ $\cot x$ *is undefined at* π *and* $-\pi$
- Asymptotes are found at π and multiples of π.



$\cot 0 = \frac{\cos 0}{\sin 0} = \frac{1}{0} = u$

$\cot \frac{\pi}{2} = \frac{\cos \frac{\pi}{2}}{\sin \frac{\pi}{2}} = \frac{0}{1} = 0$



Period: π $(0, \pi)$

input x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$y = \cot\theta$	undef	1	0	-1	undef

Domain: all real numbers except mult. of π

Range: $(-\infty, \infty)$

The same process used for sine and cosine may be followed for these trig functions. First let's look at the equations:

$$y = A\ \tan(\omega x) + B \qquad period = \frac{\pi}{\omega} \qquad y = A\cot(\omega x) + B$$

For tangent an appropriate interval is $\left(-\frac{\pi}{2\omega}, \frac{\pi}{2\omega}\right)$

for cotangent and appropriate interval is: $\left(0, \frac{\pi}{\omega}\right)$

The intervals are bounded by vertical asymptotes.

$$y = 2\tan x - 1$$

$T = \pi \qquad A = 2$

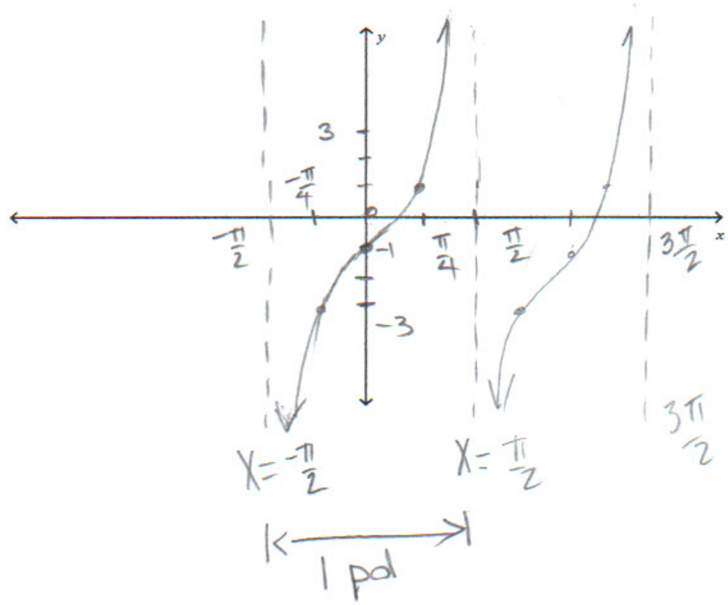
$\omega = 1$

$Pd = T = \frac{\pi}{1} = \pi$

$\underbrace{\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}_{\text{1 pd wide}}$

middle $\quad \frac{1}{2}\cdot\frac{\pi}{2}$

x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\tan x$	u	-1	0	1	u
$2\tan x$	u	-2	0	2	u
$2\tan x$ -1	u	-1 -3	-1 -1	-1 1	u



$y = 3\tan(2x)$

period = $\qquad$ $A = 3$

$\omega = 2$

pd $= \frac{\pi}{2}$

Interval

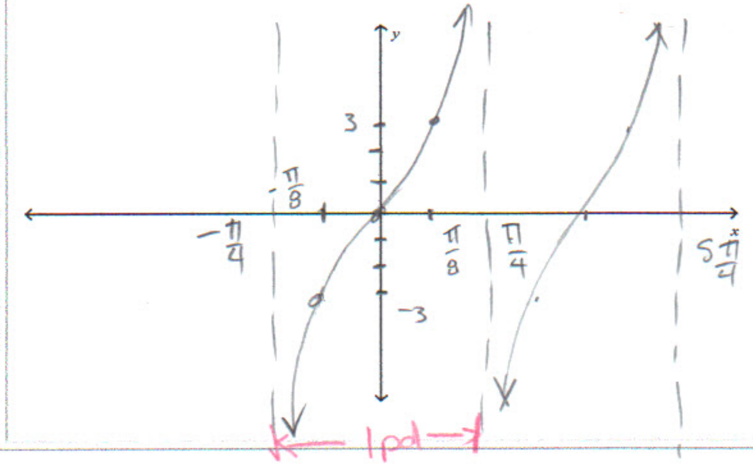
$\left(-\frac{\pi}{2(2)}, \frac{\pi}{2(2)}\right)$

$\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\frac{1}{2} \cdot \frac{\pi}{4}$

x	$-\frac{\pi}{4}\cdot 2$	$-\frac{\pi}{8}\cdot 2$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
$2x$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$ *
$\tan 2x$	u	-1	0	1	u
$3\tan 2x$	u	-3	0	3	u

* Key pts on U.C



$y = A\cot(\omega x) + B$ $\qquad$ *period* $= \frac{\pi}{\omega}$ $\qquad$ **For cotangent and appropriate interval is: $\left(0, \frac{\pi}{\omega}\right)$.**

$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

$y = \cot x + 2$

$A = 1$

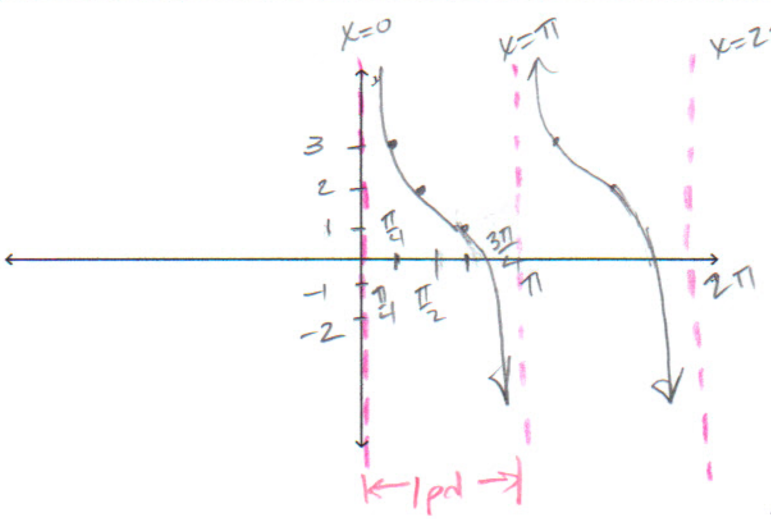
Pd $= \pi$

Interval

$\left(0, \frac{\pi}{\omega}\right)$

$(0, \pi)$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\cot x$	u	1	0	-1	u
$\cot x + 2$	+2 u	+2 3	+2 2	+2 1	u



Cosecant and Secant Functions

The cosecant and secant functions may be referred to as **reciprocal functions.** They are graphed by first graphing their reciprocals functions of sine and cosine. Where the sine and cosine functions have zeros the reciprocals of cosecant secant functions will be undefined, hence, asymptotes.

• The period for cosecant is the same as sine, 2π. • When $\sin x = 1$, the value of $\csc x = 1$. • As $\sin x \to 0.\ \csc x \to \infty\ or - \infty$ • When $\sin x = 0, \csc\ is\ undefined$ • Asymptotes at $\sin x = 0$ • graph sin	• The period for secant is the same as cosine, 2π. • When $\cos x = 1$, the value of $\sec x = 1$. • As $\cos x \to 0.\ \sec x \to \infty\ or - \infty$ • When $\cos x = 0, \sec is\ undefined$ • Asymptotes at $\operatorname{cox} x = 0$ • graph cos

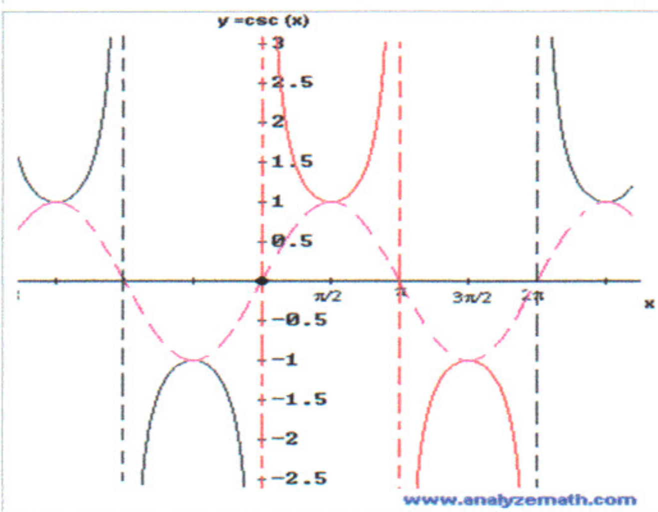
where sin x increases
csc x decreases

where cos x increases
sec x decreases

Graph $\csc\theta$ First graph $sin\ \theta$

Domain: all real numbers except multiples of π
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π

input x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{3\pi}{2}$	2π
$\boldsymbol{y = sin\ x}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0	-1	0
$\boldsymbol{y = csc\ x}$	undef	2	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	undef	-1	undef

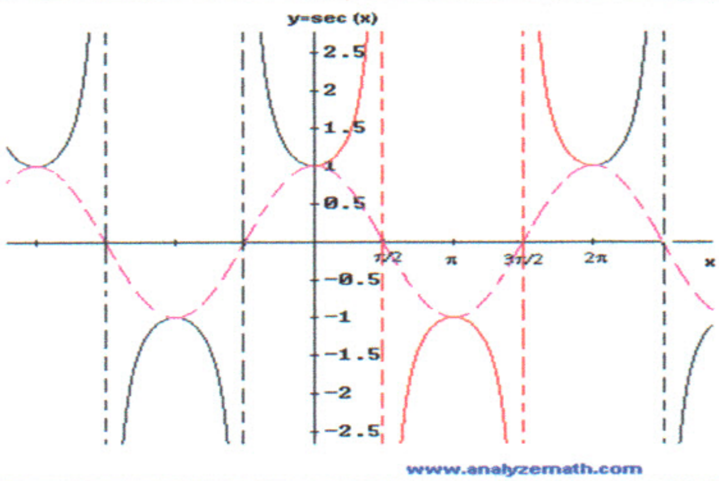


Secant is like cosecant in that it is reciprocal function. So plot the cosine function, locate vertical asymptotes at $\cos x = 0$, and graph the secant function.

Graph $sec\theta$. First plot $\cos x$.

Domain: all real numbers except odd multiples of $\frac{\pi}{2}$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π

input x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{3\pi}{2}$	2π
$y = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	-1	0	1
$y = \sec x$	1	$\frac{2\sqrt{3}}{3}$	2	u	2	-1	u	1



$y = A\csc\omega x + B$ $\qquad$ $y = A\sec\omega x + B$ $\qquad$ $period = \frac{2\pi}{\omega}$

1. graph the reciprocal function of sine or cosine, *the guide functions,*
2. Dash in the guide function.
3. Sketch vertical asymptotes; these occur at the x-values for which the guide function equals 0
4. Sketch the typical U-shaped branches, approaching the asymptotes that typify the cosecant and secant functions. Note the function's minimum is its reciprocal's maximum.

$$y = 4\csc\frac{1}{2}x$$

$T = 4\pi$ $\qquad$ $A = 4$

$\omega = \frac{1}{2}$

$Pd = \frac{2\pi}{1/2} = 4\pi$

guide, first graph : $y = 4\sin\frac{1}{2}x$

x	0	π	2π	3π	4π
$\frac{1}{2}x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\frac{1}{2}x$	0	1	0	-1	0
$4\sin\frac{1}{2}x$	0	4	0	-4	0