Precalculus
Lesson 5.4: Logarithmic Functions
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The inverse of an exponential function is a logarithmic function.

Let a be a positive number with a not equal to 1. The logarithmic function with base a is defined by:

$$\boldsymbol{log_a x = y}$$
if and only if
$$\boldsymbol{a^y = x}$$

$\log_a x = y$

$a^y = x$

Domain: $(0, \infty)$

translation: whatever you are taking the log of has to be greater than zero

Range: $(-\infty, \infty)$

start with the base and move in a counterclockwise fashion.

Write as an exponential		
$log_a 4 = 5$	$log_e b = -3$	$log_3 5 = c$
$a^5 = 4$	$e^{-3} = b$	$3^c = 5$

Write as a logarithm		
$1.2^3 = m$ (base)	$e^b = 9$	$a^4 = 24$
$\log_{1.2} m = 3$	$\log_e 9 = b$	$\log_a 24 = 4$

Find the exact value:

$log_2 16 = x$

$2^x = 16$

$2^x = 2^4$

$\underline{\underline{x = 4}}$

$log_3 \dfrac{1}{27} = x$

$3^x = \dfrac{1}{27} = \dfrac{1}{3^3}$ opposite side opposite sign

$3^x = 3^{-3}$

$\underline{x = -3}$

Find the domain of each logarithmic function:

$f(x) = log_2(x + 3)$

$x + 3 > 0$

$x > -3$

$\underline{\text{Domain } (-3, \infty)}$

$g(x) = log_5 \left(\dfrac{1+x}{1-x}\right)$

$\dfrac{1+x}{1-x} > 0$

	-2		0		2
		$(-1$		$1)$	
$1+x$	$-$		$+$		$+$
$1-x$	$+$		$+$		$-$
	$-$		$+$		$-$

Domain

$\underline{(-1, 1)}$

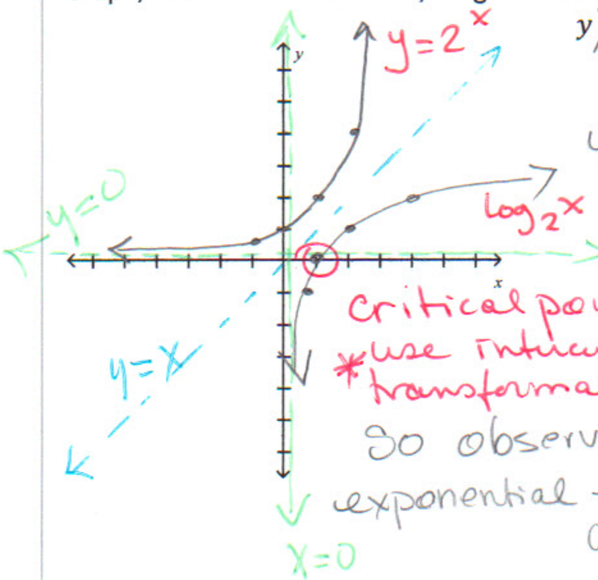
Graphing Logarithmic Functions

Knowing the general form of the graph of the log function is a short cut for graphing.

1. Write in its equivalent exponential form
2. Find the inverse; $x\ is\ y\ and\ y\ is\ x$, solve for y
3. Graph the log function's inverse, and reflect the exponential graph across the line of symmetry $y = x$.

logaritmic & exponential functions are inverses of eachother

Graph, determine the domain, range and vertical asymptote.



$y = \log_2 x \Rightarrow 2^y = x$

Inverse

$y = 2^x$

x	y
-1	1/2
0	1
1	2
2	4

flip x & y

x	y
1/2	-1
1	0
2	1
4	2

log x is y & y is x

critical point

*use intercept for transformation problems

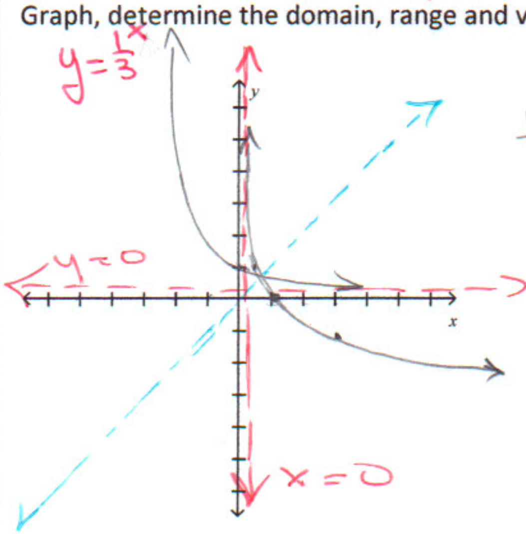
So observations:

exponential - y-intercept = log x-intercept

asymptote y=0 asymptote x=0

*use asymptote in transformation problems

Graph, determine the domain, range and vertical asymptote.



$\log_{1/3} x = y$

$\frac{1}{3}^y = x \Rightarrow y = \frac{1}{3}^x$

x	y
-2	$(\frac{1}{3})^{-2} = 9$
-1	3
0	1
1	$\frac{1}{3}$

flip for $\log_{\frac{1}{3}} x$

x	y
9	-2
3	-1
1	0
$\frac{1}{3}$	1

asymptote = $x = 0$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

The Natural and Common Logarithm

The Natural Logarithm is a logarithm with the base e. It is written with the abbreviation of ln.

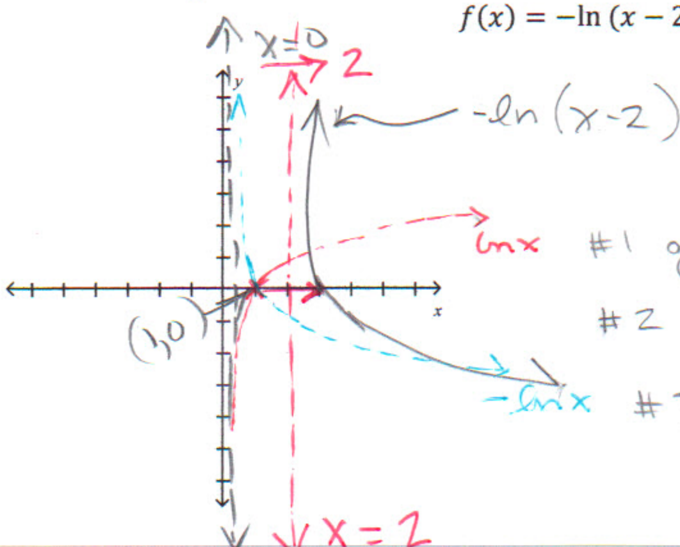
$$y = \ln x$$
$$\text{if and only if} \qquad x = e^y$$

The logarithm with base 10 is called the **common logarithm** and is denoted by omitting the base:

$$\log x = log_{10}x$$

Graph, determine the domain, range and vertical asymptote. Identify the inverse and the domain and range of the inverse.

$$f(x) = -\ln(x-2)$$



transformations?

"–" reflect across x-axis

"-2" → right 2

#1 graph ln x = critical point

#2 reflect over x-axis

#3 move right 2 units
move asymptote & critical point

Solving Logarithmic Equations – write as exponential

$log_3(4x-7) = 2$	$log_x 64 = 2$
$3^2 = 4x - 7$	$x^2 = 64$
$9 = 4x - 7$	$x^2 = 8^2$
$\frac{16}{4} = \frac{4x}{4}$	exponents equal
	bases equal
$\underline{4 = x}$	$\therefore\ \underline{x = 8}$

Using Logarithms to Solve and Exponential Equation write as log.

$$e^{2x} = 5$$

$\log_e 5 = 2x$

$\ln 5 = 2x$

$\frac{\ln 5}{2} = x$

$.805 \approx x$