Algebra II

Lesson 3: Transformation Rules for Algebraic Equations

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When a number is added or changed in an algebraic equation, a transformation will occur. The graph will be moved up or down, left or right, or be stretched or shrunk. These changes are known as **transformations.** When a parent function $f(x)$ is transformed, it becomes a different function. Let's use $t(x)$ for the transformed function.

$f(x+h)$	move the x - *value* $\boldsymbol{h}$ *units left*	graph will slide horizontally left
$f(x-h)$	move the x- *value* $\boldsymbol{h}$ *units right*	graph will slide horizontally right
$a(f(x))$	multiply the y- *values by* $\boldsymbol{a}$	$a > 0$ *vertical stretch/ steeper or narrower* $0 < a < 1$ *fraction* *vertical shrink/flatter or wider*
$-f(x)$	graph will flip upside down	Reflection across x-axis
$f(x)+k$	move y- *value* $\boldsymbol{k}$ *units up*	Vertical translation up k units
$f(x)-k$	move y- *value* $\boldsymbol{k}$ *units down*	graph will slide vertically down k units
Put it all together	$\boldsymbol{t(x) = a \cdot f(x-h)+k}$	

Enter parent function into calculator, and then enter equation below, what happened?

1. $t(x) = 2x^2$ Stretch factor 2 (2)(y)	$t(x) = \frac{1}{4}x^2$ shrink factor 1/4 (1/4)(y)	$t(x) = x^2 + 3$ ↑ 3	$t(x) = -x^2 + 3$ Reflect over x-axis ↑ 3
2. $t(x) = x^2 - 3$ ↓ 3	$t(x) = (x-3)^2$ → 3	$(x) = (x^2+3)$ ← 3	$(x) = -(x^2+3)$ Reflect over x-axis ← 3
3. $t(x) = 2\|x\|$ stretch factor (2)(y)	$t(x) = \|x+2\|$ ← 2	$t(x) = \|x\| + 2$ ↑ 2	$t(x) = -\|x\| + 2$ reflect over x-axis ↑ 2
4. $t(x) = \frac{5}{x}$ Stretch factor 5 (5)(y)	$t(x) = \frac{1}{x+4}$ ← 4	$t(x) = \frac{1}{x} + 3$ ↑ 3	$t(x) = -\left(\frac{1}{x}+3\right)$ Reflect over x-axis ← 3

The order of operations for transformations is similar to those of equations; we deal with the **multiplication** before **addition/subtraction.** For graphs of functions involving more than one transformation, apply each change in the following order::

1. Horizontal Translation
2. Stretching or shrinking
3. Reflection
4. Vertical shift up/down

Examle 1... Describe each combined transformation, *in the correct order*.

a. $f(x-2)+3$

→ 2

↑ 3

b. $\frac{1}{2}\,g(x)+3$

Shrink: $\left(\frac{1}{2}\right)(y)$

↑ 3

c. $-2g(x)-7$

reflect over x-axis

Stretch: 2•y

↓ 7

d. $3h(x-4)+1$

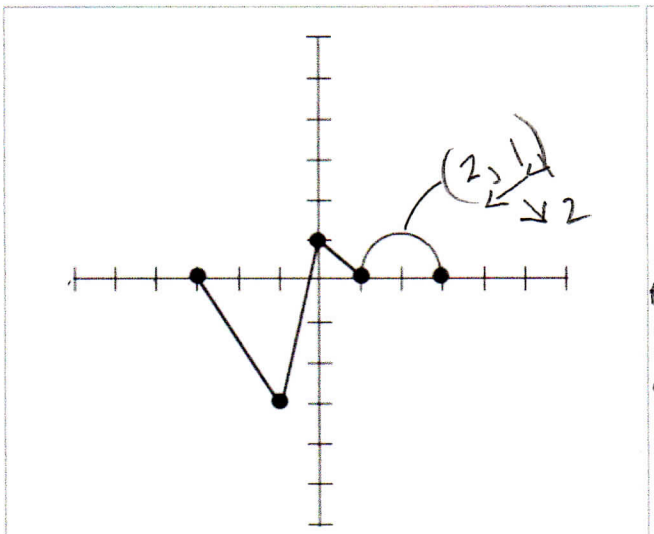
→ 4

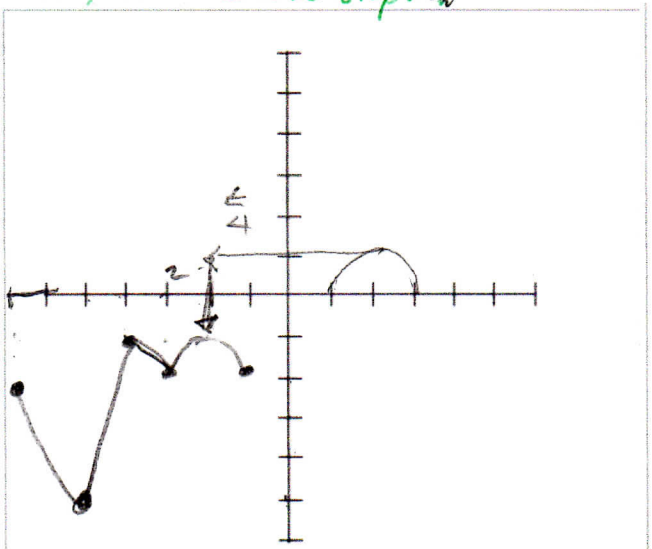
Stretch 3•y

↑ 1

When dealing with just a graph of a function, look at the x-y ordered pairs. For a horizontal shift, work with the x-value. For the stretch and vertical translation work with the y-value.

Example 2 Transform the function below to $h(x+4)-2$? Show each step!!!





$(-3,0)$	$(-1,-3)$	$(0,1)$	$(1,0)$	$(3,0)$
← 4	← 4	← 4	← 4	← 4
↓ 2	↓ 2	↓ 2	↓ 2	↓ 2
$x: -3 \leftarrow 4$	$x: -1 \leftarrow 4$	$x: 0 \leftarrow 4$	$x: 1 \leftarrow 4$	$3 \leftarrow 4$
$x = -7$	$x = -5$	$x = -4$	$x = -3$	$x = -1$
$y: 0 \downarrow 2$	$y: -3 \downarrow 2$	$y: 1 \downarrow 2$	$0 \downarrow 2$	$0 \downarrow 2$
$y = -2$	$y = -5$	$y = -1$	-2	-2
$(-7,-2)$	$(-5,-5)$	$(-4,-1)$	$(-3,-2)$	$(-1,-2)$