Lesson 9-1
Inverse Variation

When the ratio of two variables has a constant (unchanged) ratio, their relationship is called a **direct variation**. We say that y varies directly as x. The constant ratio, **k**, is called the **constant of variation**.

$$\frac{y}{x} = k, \qquad or \ \ y = kx$$

Note: in a linear situation the constant of variation is our slope:

$$\frac{rise}{run} = \frac{y}{x}$$

In direct variation problems, we will see that as one variable increases the other increases. Likewise as one decreases so will the other decrease. varies directly ⇒ write $= k$

$= k$

Melissa's weekly salary, **s**, varies directly as the number of hours, **h**, that she works. Write an equation that describes this relation. Solve for the constant of variation.
Melissa's check shows she worked 32 hours and it is the amount of $363.20. What is her hourly rate?

$s = kh$

$k = \frac{s}{h}$

$k = \frac{363.20}{32\text{ hr}} = \$11.35/\text{hr}$

? varies directly $= k$

According to Hooke's Law, the force needed to stretch a spring is proportional to the amount the spring is stretched. If fifty pounds of force stretches a spring five inches, how much will the spring be stretched by a force of 120 pounds?

$F = kd$

$50 = k5$

$\frac{50}{5} = 10 = k$

$120 = 10d$

$\frac{120}{10} = 12$ inches

If y varies directly as x^2 and $y = 8$ when $x = 2$, find y when $x = 1$. Write the equation of variation.

$y = kx^2$

$8 = k(4)$

$\frac{8}{4} = 2$

$y = 2x^2$

$y = 2(1^2)$

$y = 2$

inverse

The opposite of direct variation is known as **Inverse Variation**. In an inverse variation, the values of the two variables change in an opposite manner, that is, as one value increases, the other decreases.

y varies inversely as x

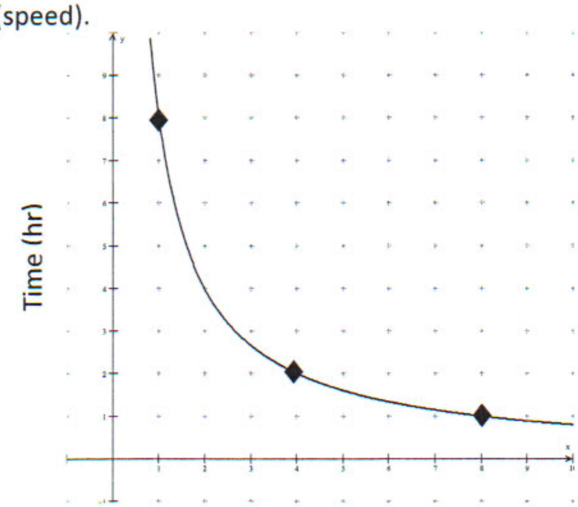
$$xy = k, \quad so\ y = \frac{k}{x}, \quad or\ x = \frac{k}{y}$$

Let's look at an example. How long will it take a cycler to bike 8 miles? Well that depends on his speed. A biker traveling at 8 mph can cover 8 miles in 1 hour. If the biker's speed decreases to 4 mph, it will take the biker 2 hours to cover the same distance.

Rate (mi/hr)	Time (hr)
8	1
4	2
1	8

Notice that as the rate decreases, the time increases. Cut the rate in half, the time doubles. Our rate equation may be written as $t = \frac{d}{r}$, at distance equal to 8 miles we get: $t = \frac{8}{r}$

Graphically we see can see the relation between time and rate (speed).



Given y varies inversely as x. Write a variation function when $y = 1.4\ and\ x = 0.3$
So: $k = ?$ and our inverse variation function is?

$y = \frac{k}{x}$ or

$yx = k$

$1.4(.3) = k = .42$

$y = \frac{.42}{x}$

$\frac{y}{x} = k$ $xy = k$

Determine if the relationship between the values is direct variation, inverse variation or neither. Write an equation if possible.

x	2	4	6
y	3.2	1.6	1.1

neither

$k \overset{?}{=} xy = 6.4$ 6.4 6.6

x	0.8	0.6	0.4
y	0.9	1.2	1.8

Inverse Variation

$k = xy$ $7.2 = k$ 7.2 7.2 constant

$y = \frac{7.2}{x}$

x	2	5	8	9.5
y	14	35	56	66.5

Direct Variation

$\frac{y}{x}$ 7 7 7 7 constant

$y = 7x$

x	2	2.5	5	6
y	30	24	12	10

Inverse Variation

$k = xy = 60$ 60 60 60

constant!

$y = 60/x$

Write the function that models each inverse variation. Then find $y\ when\ x = 9$.

$x = 3\ when\ y = -5$

$xy = k$

$3(-5) = k = -15$

$y = -15/x$

$y = -15/9$

$y \approx 1.67$

The inverse variation contains the ordered pair: $(6, 3)$

$xy = k = (6)(3) = 18$

$y = 18/x$

$y = 18/9 = 2$

$y = 2$

Combined Variation

Variation is not only for linear relationships. We can just as easily have a situation where y varies inversely with x^2, such that: $y = \frac{k}{x^2}$. Also, we can have situations where we have what is called a **joint variation**. Here a variable will vary jointly with two other variables: $z = kxy$. Let's put some of these combinations into table form:

Combined Variation	Equation form
z varies jointly with *x* and *y*. (directly with both x & y)	$z = kxy$
z varies jointly with *x* and *y* and inversely with *w* (w in denominator)	$z = \frac{kxy}{w}$
z varies directly with *x* and inversely with the product *wy*	$z = \frac{kx}{wy}$

$=k$

Given that z varies directly with x and inversely with y. Write a variation function when $x = 6, y = 2, and\ z = 15$

$z = \frac{kx}{y}$ => $15 = \frac{k6}{2}$

$15 = 3k$

$15/3 = k = 5$

$z = \frac{5x}{y}$

Given that z varies jointly with x and y. Write a variation function when $x = 2,\ y = 3\ and\ z = 60$

$z = kxy$

$60 = k(2)(3)$

$60 = 6k$

$60/6 = k = 10$

$z = 10xy$

Describe the combined variation that is modeled by each formula:

a) $A = \pi r^2$

$=k$

Area varies directly with the square of the radius

b) $h = \frac{2A}{b}$ direct

inverse

height varies directly with the area and inversley with the base

The volume **V** of a tetrahedron varies jointly with its altitude **h** and base of area **B**. Find the formula that models this joint variation. Given that the tetrahedron has an altitude of 5 cm., a base area of 6 cm^2, and a volume of 10 cm^3

V h B

$V = khB$

$10 = k(5)(6)$

$10 = 30k$

$\frac{10}{30} = \frac{1}{3} = k$

=> $V = \frac{1}{3}hB$

Other stuff

Given a direct variation, find the missing variable for the pair of values: $(4,6)$, $(x,3)$

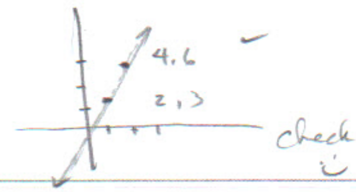
$y = kx$

$\frac{y}{x} = k = \frac{6}{4} = \frac{3}{2}$

$\left(\frac{2}{3}\right)3 = \frac{3}{2}(x)\left(\frac{2}{3}\right)$

$2 = x$

$\Rightarrow (2,3)$



Given an inverse variation, find the missing variable for the pair of values: $(4,6)$, $(x,3)$

$y = \frac{k}{x}$

$xy = k$

$4(6) = k = 24$

$3 = \frac{24}{x}$

$x = 24/3 = 8 = x$

$\Rightarrow (8,3)$