Notes: Rational Equation Word Problems

Rational Equations are used often in problems relating to rate & time – or "work" problems. These problems typically involve two people or machines completing a certain task. We can use the following basic setup to solve these types of problems:

$$\text{rate} = \frac{\text{"job"}}{\text{time}} \qquad \text{rate 1} + \text{rate 2} = \text{combined rate}$$

Example 1... If two inlet pipes are both open, they can fill a pool in 1 hour and 12 minutes. One of the pipes can fill the pool by itself in 2 hours. How long would it take the other pipe to fill the pool by itself?

| | Time | rate |
|---|---|---|
| pipe 1 | 2hr | 1 pool/2hr |
| pipe 2 | t | 1 pool/t hr |
| together | 1hr 12min = 1.2hr | 1pool/1.2hr |

$\rightarrow 12\text{min}\left(\frac{1\text{hr}}{60\text{min}}\right) = .2\text{hr}$

$$\frac{t}{t}\cdot\frac{1}{2} + \frac{1}{t}\cdot\frac{2}{2} = \frac{1}{1.2}$$

$$\frac{t+2}{2t} = \frac{1}{1.2}$$

$$1.2(t+2) = 2t$$

$$1.2t + 2.4 = 2t$$

$$2.4 = .8t$$

$$2.4/.8 = \boxed{3\text{ hr to fill pool}}$$ 2nd pipe

Example 2... Two Algebra 2 students, Frank and Jill, decide to do their review together to save time. If Frank could finish his review in 3 hours, and Jill could do hers in 2 hours and 15 minutes, how long would it take them to do the review if they work together?

| | time | rate |
|---|---|---|
| Frank | 3hr | 1review/3hr |
| Jill | 2hr 15min | 1 review/2.25hr |
| together | t hr | 1review/t hr |

$$\frac{1}{3} + \frac{1}{2.25} = \frac{1}{t}$$

$$.\overline{7} \approx \frac{7}{9} = \frac{1}{t}$$

$$9 = 7t$$

$$\frac{9}{7} = t = \boxed{1.28\text{ hr if they work together}}$$

Example 3... You and your friend work together doing lawn work for your neighbors. Usually you work together, and it takes an hour and a half to mow & trim one of your neighbor's lawn. Your friend has done it by himself before, and it takes him 2 hours to do by himself. How long would it take you to mow and trim the lawn by yourself?

| | |
|---|---|
| me | t |
| friend | 2hr |
| together | 1.5hr |

$$\frac{1}{t} + \frac{1}{2} = \frac{1}{1.5}$$

$$\frac{1}{t} = \frac{1}{1.5} - \frac{1}{2} = .16\overline{6}$$

$$\frac{1}{t} = \frac{1}{6}$$

$$t = \boxed{6\text{ hours for me to mow \& trim}}$$

Another type of problem that uses rational expressions are distance/rate/time problems. For these types of problems, it can help to use a table to hold all our information. Just remember that distance always equals rate times time.

Example 4... To travel 60 miles, it takes Sue, riding a moped, 2 hours less time than it takes Doreen to travel 50 miles riding a bicycle. Sue travels 10 miles per hour faster than Doreen. Find the rates and times of both girls.

| | distance | rate | time |
|---|---|---|---|
| Sue | 60 mi | $10+r$ mph | $t-2$ hr |
| Doreen | 50 mi | $r$ mph | $t$ hr |

$r = d/t$

Doreen $r = \frac{50}{t}$

Substitute

Sue $10 + r = \frac{60}{t-2}$

$$10 + \frac{50}{t} = \frac{60}{t-2}$$

$$(t-2)(t)10 = \left(\frac{60}{\cancel{t-2}} - \frac{50}{\cancel{t}}\right)(t-2)(t)$$

$$10t(t-2) = 60t - 50(t-2)$$

$$10t^2 - 20t = 60t - 50t + 100$$

$$10t^2 - 30t - 100 = 0$$

$$\cancel{10}(t^2 - 3t - 10) = 0$$

$$(t-5)(t+2) = 0$$

$$t = 5 \text{ or } \cancel{-2}$$ neg. time?

Doreen:
5 hours time

Example 5... Julian's boat will go 15 miles per hour in still water. If he can go 12 miles downstream in the same amount of time as it takes him to go 9 miles upstream, then what is the speed of the current?

| | distance | rate Boat & current | time |
|---|---|---|---|
| downstream | 12 mi With current | $15+r$ | $t$ |
| upstream | 9 mi, against current | $15-r$ | $t$ |

equal

$r = \frac{d}{t} \rightarrow t = \frac{d}{r}$

$\downarrow t = \frac{12}{15+r}$

$\uparrow t = \frac{9}{15-r}$

$$\frac{12}{15+r} = \frac{9}{15-r}$$

$$9(15+r) = 12(15-r)$$

$$135 + 9r = 180 - 12r$$

$$21r = 45$$

$r = 2.14$ mi/hr stream current