Algebra II
Lesson 9.4: Rational Expressions
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When given a fraction like $\frac{3}{27}$ we can simplify it by reducing, that is taking out the common factors in the form of $\frac{1}{1}$ $or\ here\frac{3}{3}$ so: $\frac{3}{27}=\frac{1}{9}$ Well, you have probably figured out where this is going; those rational functions from the previous section may also be reduced by taking out common factors.

Factors!!
A number may be made by multiplying two or more other numbers together. The numbers that are multiplied together are called factors of the final number.

In a polynomial, we also factor by separating the polynomial into polynomials of lesser degree that multiply out to be equivalent to the original polynomial. .

When dealing with a RATIONAL POLYNOMIAL, you can only cancel out common factors when you have factors that are in common with both the numerator and denominator. ***ALWAYS FACTOR before cancelling out what you think to be a common factor.***

Examples: $\frac{3(x+2)(x-3)}{(x+2)}$ $\therefore = 3(x-3)$	**Step 1** Factor the numerator and denominator (if needed) into a product of smaller factors. Are there any common factors? **Step 2 :** Anything divided by itself is "1", so factor out. Reduce and rewrite.
$\frac{x^2+x-12}{x^2+5x-24}$	$\frac{-27x^3y}{9x^4y}$
$\frac{x^2+10x+25}{x^2+9x+20}$	$\frac{2x^2-8x-64}{x^3+10x^2+24x}$

RESTRICTIONS! When there are variables in the denominator, there are numbers that if the variable is equal, would cause the denominator to be equal to zero. So always define (state) the restrictions. *Even when the variable is cancelled out with a common factor in the numerator, the restriction still holds true. Look at the original problem.*

State restrictions on the examples above.

Warning: The common temptation at this point is to try to cancel out common terms. *NEVER EVER NEVER* cancel out terms that are either added or subtracted. Whenever you have terms added together, there are invisible parentheses around them, like this: $\frac{(x+4)}{(x+8)}$. You can only cancel out factors (that is, entire expressions contained within parentheses), not terms that are separated by arithmetic signs.

Think: $\frac{4+9}{1+9} = \frac{13}{10} = 1.3$ **NOT!!!!** $\frac{4+\cancel{9}}{1+\cancel{9}} = \frac{4}{1} = 4$ ***IT IS ILLEGAL TO CANCEL OUT PARTS OF THE TERMS!! YOU WOULD NOT DO THIS, THEREFORE DO NOT DO THIS WITH RATIONAL EXPRESSIONS!*** **SO NEVER DO THE FOLLOWING!** $\frac{\cancel{x}+3}{\cancel{x}+6} = \frac{3}{6} = \frac{1}{2}$

Multiplication of Rational Expressions

What is possible with a simple numerical fraction is also possible with rational expressions. As with fractions, multiplication will be easier if you simplify first then multiply:

$\frac{\cancel{12}}{\cancel{8}} \cdot \frac{\cancel{64}}{\cancel{36}} = \frac{1}{1} \cdot \frac{8}{3} = \frac{8}{3}$ factor out an 8 and a 12. by the time you reduce, you have nothing to multiply!

$\frac{2x^2+7x+3}{x-4} \cdot \frac{x^2-16}{x^2+8x+15}$	$\frac{a^2-4}{a^2-1} \cdot \frac{a+1}{a^2+2a}$

Remember the long hand way you learned to divide back in elementary school? *Switch it and flip it!*

$\frac{5}{8} \div \frac{3}{4} = \frac{5}{\cancel{8}} \times \frac{\cancel{4}}{3} = \frac{5}{2} \times \frac{1}{3} = \frac{5}{6}$

We can follow the same process for division with rational expressions. *NOTE: Any restrictions on the variables also go for the divisor's numerator! Look below?*

$\frac{a^2+2a-15}{a^2-16} \div \frac{a+1}{3a-12}$	$\frac{(4-x)}{(3x+2)(x-2)} \div \frac{5(x-4)}{(x-2)(7x-5)}$