Lesson 9-1
Inverse Variation

When the ratio of two variables has a constant (unchanged) ratio, their relationship is called a **direct variation**. We say that y varies directly as x. The constant ratio, **k**, is called the **constant of variation**.

$$\frac{y}{x} = k, \qquad or\ y = kx$$

Note: in a linear situation the constant of variation is our slope:

$$\frac{rise}{run} = \frac{y}{x}$$

In direct variation problems, we will see that as one variable increases the other increases. Likewise as one decreases so will the other decrease.

Melissa's weekly salary, **s**, varies directly as the number of hours, **h**, that she works. Write an equation that describes this relation. Solve for the constant of variation.
Melissa's check shows she worked 32 hours and it is the amount of $363.20. What is her hourly rate?

According to Hooke's Law, the force needed to stretch a spring is proportional to the amount the spring is stretched. If fifty pounds of force stretches a spring five inches, how much will the spring be stretched by a force of 120 pounds?

If y varies directly as $x^{2,} and\ y = 8\ when\ x = 2,\ find\ y\ when\ x = 1.$ Write the equation of variation.

The opposite of direct variation is known as **Inverse Variation**. In an inverse variation, the values of the two variables change in an opposite manner, that is, as one value increases, the other decreases.

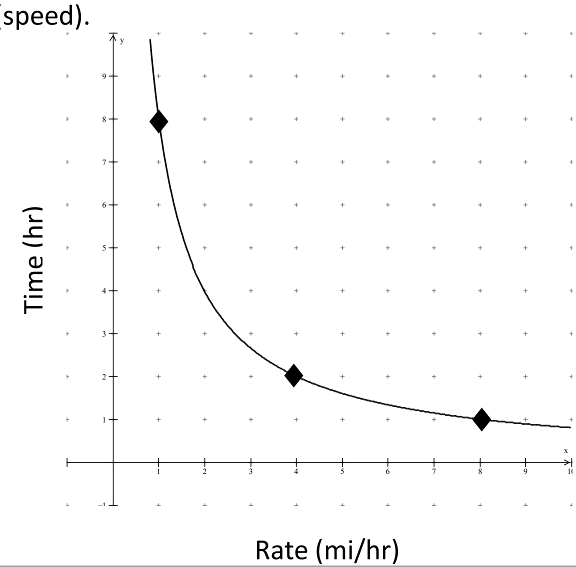
$$xy = k, \qquad so\ \ y = \frac{k}{x}, \qquad or\ \ x = \frac{k}{y}$$

Let's look at an example. How long will it take a cycler to bike 8 miles? Well that depends on his speed. A biker traveling at 8 mph can cover 8 miles in 1 hour. If the biker's speed decreases to 4 mph, it will take the biker 2 hours to cover the same distance.

Rate (mi/hr)	Time (hr)
8	1
4	2
1	8

Notice that as the rate decreases, the time increases. Cut the rate in half, the time doubles. Our rate equation may be written as $t = \frac{d}{r}$, at distance equal to 8 miles we get: $t = \frac{8}{r}$

Graphically we see can see the relation between time and rate (speed).



Given y varies inversely as x. Write a variation function when $y = 1.4\ and\ x = 0.3$
So: $k = ?$ and our inverse variation function is?

Determine if the relationship between the values is direct variation, inverse variation or neither. Write an equation if possible.

x	2	4	6
y	3.2	1.6	1.1

x	0. 8	0.6	0.4
y	0.9	1.2	1.8

x	2	5	8	9.5
y	14	35	56	66.5

x	2	2.5	5	6
y	30	24	12	10

Write the function that models each inverse variation. Then find $y \text{ when } x = 9$.

$x = 3 \text{ when } y = -5$	The inverse variation contains the ordered pair: $(6, 3)$

Combined Variation

Variation is not only for linear relationships. We can just as easily have a situation where y varies inversely with x^2, such that: $y = \frac{k}{x^2}$. Also, we can have situations where we have what is called a **joint variation**. Here a variable will vary jointly with two other variables: $z = kxy$. Let's put some of these combinations into table form:

Combined Variation	Equation form
z varies jointly with *x* and *y*	$z = kxy$
z varies jointly with *x* and *y* and inversely with *w*	$z = \frac{kxy}{w}$
z varies directly with *x* and inversely with the product *wy*	$z = \frac{kx}{wy}$

Given that z varies directly with x and inversely with y. Write a variation function when $x = 6, y = 2, and\ z = 15$

Given that z varies jointly with x and y. Write a variation function when $x = 2,\ y = 3\ and\ z = 60$

Describe the combined variation that is modeled by each formula:

a) $A = \pi r^2$

b) $h = \frac{2A}{b}$

The vol*u*me **V** of a tetrahedron varies jointly with its altitude **h** and base of area **b**. Find the formula that models this joint variation. Given that the tetrahedron has an altitude of 5 cm., a base area of 6 cm^2, and a volume of 10 cm^3

Other stuff

Given a direct variation, find the missing variable for the pair of values: $(4,6), (x, 3)$

Given an inverse variation, find the missing variable for the pair of values: $(4,6), (x, 3)$