Algebra II

Lesson 8-2.A: Models of Exponential Functions

MONEY !!

When you put money into savings at a bank or an investment firm, the institute pays you money for letting them use your money. This is called **interest.** They in turn use your money to loan out to someone else who needs to borrow money to buy a house, car, or for college expenses, etc.

Compounded Interest

One method used to determine interest is to compound or calculate it periodically a certain number of times per year such as annual: 1 time, semi-annual: 2x, quarterly: 4x, or even monthly: 12x.

In these cases the exponential function, $y = ab^x$ is:

:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

n is the number of times per year the interest is compounded

How much money will you have if you invest $2000 (P) for 3 years (t) with an annual interest rate of 5.1% (r) compounded quarterly (4)? After 25 years (t)?

.051

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 2000\left(1+\frac{.051}{4}\right)^{4(3)}$$

$$= 2000(1.01275)^{12}$$

$$= 2328.39693$$

Money = $2328.40

$$A = 2000\left(1+\frac{.051}{4}\right)^{4(25)}$$

$$= 2000\left(1+\frac{.051}{4}\right)^{100}$$

$$= \$7099.95$$

Continuously compounded interest

Next class we will look at a special irrational number known as Euler's number, **e.** In finances, **e** is used in calculating interest earned. Interest earned on investments and applied to loans often is calculated using a method of continuously compounding the accrued interest. Our exponential function form $y = ab^x$ can now be written as:

$$A = Pe^{rt}$$

A= amount (final)

P=Principal amount

r=rate of interest (annual)

t=time in years

Calculator

$\boxed{2nd}$ $\boxed{\div}$

~ 2.72

How much money will you have if you invest \$2000 for 3 years with an annual interest rate of 5.1% continuously compounded? After 25 years?

P t r

$A = Pe^{rt}$ .051 r

$A = 2000e^{.051(3)}$

$= \$2330.65$ (a bit more than quarterly compound)

$e^{\square}$

2nd LN

$A = 2000e^{(.051)(25)}$

$= \$7157.40$

Read carefully

e

A student wants to save \$8000 for college in five years. How much should be put into an account that earns 5.2% annual interest compounded continuously?

A t e r = .052

$A = Pe^{rt}$

$$\frac{8000}{e^{(.052)(5)}} = \frac{Pe^{(.052)(5)}}{e^{(.052)(5)}}$$

$$\frac{8000}{e^{(.052)(5)}} = P = \$6168.41$$

Half-Life

Half-life is a method that may be used to determine the age of substances. Half-life is the time it takes for half of an original material to decay or change into another substance. Our exponential equation now becomes:

$$\boldsymbol{A = A_0\left(\frac{1}{2}\right)^{t/k}}$$

b

now dividing time by half-life time

A = amount of remaining material

A_0 = initial amount

the rate of decay = ½ $= r$ $= .5$

t=time

k=half-life time

$b = 1 - r$

$= 1 - .5$

$b = .5 = \frac{1}{2}$

Technetium-99m has a half-life of 6 hours. Find the amount of technetium-99m that remains from a 50-mg supply after 25 hours.

k A_0 t

$$A = A_0\left(\tfrac{1}{2}\right)^{t/k}$$

$A_0 = 50$
$t = 25$
$K = 6$

$$A = 50\left(\tfrac{1}{2}\right)^{\frac{25}{6}}$$

$$A = 2.18 \text{ mg}$$

units

Arsenic-74 is used to locate brain tumors. It has a half-life of 17.5 days. Write an exponential decay function for a 90-mg sample. Use the function to find the amount remaining after 6 days

k A_0 t

$$A = 90\left(\tfrac{1}{2}\right)^{6/17.5}$$

$$A = 10.96\text{mg}$$

$A_0 = 90$
$t = 6$
$k = 17.5$

Seismology
The magnitude of an earthquake is a measure of the amount of the energy released t its source. The Richter scale is an exponential measure of earthquake magnitude. The energy released by an earthquake can be represented by the expression: $\boldsymbol{E \cdot 30^x}$, *where x is the magnitude on the Richter Scale*.

In 1995, an earthquake in Mexico registered 8.0 on the Richter scale. In 2001, an earthquake of magnitude 6.8 shook Washington state. Compare the amounts of energy released in the two earthquakes.

x x ratio

$$\frac{\cancel{E} \times 30^{8}}{\cancel{E} \times 30^{6.8}} = 59.2$$

Means 59.2 times more energy

Compare the amounts of energy released in the 1995, Mexico earthquake that registered 8.0 on the Richter scale to the March 1964 earthquake in Alaska that registered 9.2 on the Richter Scale.

$$\frac{\cancel{E} \times 30^{8}}{\cancel{E} \times 30^{9.2}} = .0168 \text{ times less in Mexico}$$

OR flip $\Rightarrow \frac{30^{9.2}}{30^{8}} = 59.2$ times more in Alaska

NOTE on *e* and natural logarithms. We will be looking at logarithms in the upcoming sections. Please note that as we had inverse functions we also have in inverse of ***e*** – the ***natural logarithm***. When given a problem that requires you to solve for the time or interest rate, take the natural log of both sides (this is the inverse):

$2x = xe^{.08t}$ first note that the x's cancel out

$ln\, 2 = lne^{.08t}$ take *ln* of each side

$.6931 = .08t$ sove for t by dividing through by the reciprocal of .08

$.6931/_{.08} = 8.7 = t$