Algebra II

Lesson 8.1 Exploring Exponential Models

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Exponential functions are similar in looks to our other functions involving exponents, but there is a big difference. The variable is now the power, rather than the base. **Exponential functions** are used to predict population growth patterns, nuclear power reactions, ozone and rain forest depletion. There are two types of exponential functions:

1. **Growth** function, which means *as **x increases, y increases exponentially*** *(* the graph will get larger).
2. **Decay** function, which means *as **x increases, y decreases exponentially*** (the graph will get smaller).

An exponential function has the form:

$$f(x) = ab^x$$

$\boldsymbol{where\ a\ is\ a\ coefficient},\ \ \boldsymbol{b}\ (\boldsymbol{base}) > 0, b \neq 1, \qquad and\ x\ is\ an\ exponent$

- If $0 < b < 1$, then you will have a decay function
- If $b > 1$, then you will have a growth function.
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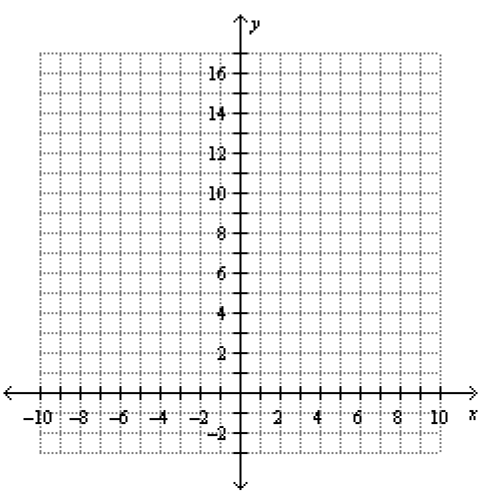
GROWTH

- Base >1
- graph rises as x-values get larger

Graph $y = 2^x$

x	$2^x \qquad = y$
-3	$2^{-3} = 8^{-1} = \frac{1}{8}$
-2	
-1	
0	
1	
2	
3	

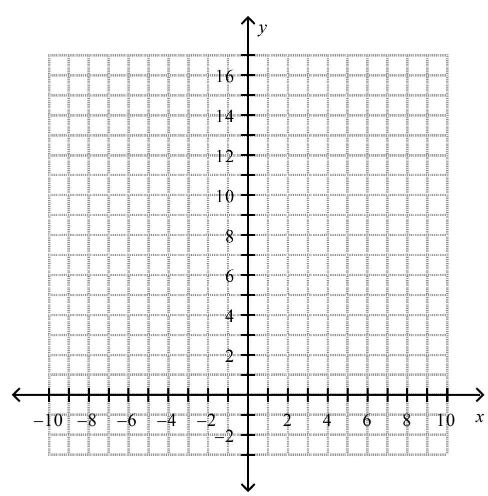


DECAY

- 0<base <1
- graph falls as x-values get larger

Graph $y = \left(\frac{1}{2}\right)^x$

x	$\left(\frac{1}{2}\right)^x \qquad = y$
-3	$\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$
-2	
-1	
0	
1	
2	
3	



Notice that the graphs of both types of exponentials get infinitely close to but **do not touch or cross** the x-axis.

Asymptote – a line that a graph approaches as *x or y increases* in absolute value. It is like an invisible boundary that a graph cannot cross or touch.

Rate of increase/decrease

The exponential form $f(x) = ab^x$ is revised to calculate growth or decay patterns, given a fixed rate of increase (growth) or decrease (decay).

$$\boldsymbol{A(t) = a(1 \pm r)^t},$$

- **a** is the initial amount
- **A(t)** is the final amount at time **t**
- **the base** is $1 \pm r$
- **r** is the rate of increase, so write the base as: $(1 + r)$
- **if** decrease, write the base as: $(1 - r)$
- **t** is the number of time periods

- $if\ \ r = 15\%, we\ write\ \ r\ as\ 0.15$

Identify the functions as growth or decay, what is the rate of increase/decrease?

$y = 100(0.12)^x$	$f(x) = 0.2(1.74)^x$
$h(x) = 16(3.4)^x$	$y = 32\left(\frac{14}{10}\right)^x$

Exponential functions are used to calculate compound interest, bacteria colony growth, human population patterns, appreciation (increased value) and depreciation (decreased value) of cars, truck homes or office buildings, and many other similar applications.

Depreciation of a new car

A new car costs about \$24,500. It is estimated that the car will depreciate (lose value) by 15% each year. What will the car be worth in 4 years?

An abandoned house has a mouse population of 22. It is increasing at a rate of 5% per month. Write a function that models the population. Estimate when there will be 50 mice in the house.

Reality check: Try a rate of growth of115% and time to get a population of 1000 of the little guys.

With a couple points from an exponential graph, we can calculate an exponential function that models the data:

Write an exponential function in the form $\boldsymbol{y = ab^x}$ for the graph that includes the points: $(1,6) and\ (0,2)$

What is the horizontal asymptote of this function?

Is this an exponential increase or decrease function?

1. Using the general form, substitute (0,2) for x and y. Recommend to use the point with the smallest x-value first.
2. Solve for **a**
3. Using the exponential equation again, substitute the calculated value of **a** and (1,6) for x and y.
4. With the division property of exponents, simplify and solve for b
5. With a solution for **b** go back to our equation for **a=,** and solve for **a**
6. With **a and b** we can now write the exponential equation for a curve that contains the given points.

Write an exponential function $y = ab^x$ for a graph that includes the points:

Given two points $(2,4)\ and\ (3,16)$, from an exponential graph, determine the exponential function.	$(-3, 24) and\ (-2, 12)$