Examples to accompany Applications of Logarithmic Functions Worksheet:

The concentration of hydrogen ions in a substance determines how acidic it is. The greater the concentration hydrogen ions, the more acidic. The pH and number of hydrogen ions are related in the formula: *pH = -log [H⁺]*

Example 1... The pH of milk is 6.6, while the pH of seawater is 8.5. Find the concentration of hydrogen ions in each substance. Which substance is more acidic?

Milk: $ph = -\log[H^+]$

$(-1)6.6 = -\log[H^+](-1)$

$-6.6 = \log_{10}[H^+]$

$10^{-6.6} = H^+$

$2.51 \times 10^{-7} \approx H^+$ milk

greater concentration: more acidic

Seawater:

$8.5 = -\log[H^+]$

$-8.5 = \log H$

$10^{-8.5} = H^+$

3.16×10^{-9} seawater H^+

Example 2... The U.S. population of peninsular bighorn sheep was 1,170 in 1971. By 1999, only 335 remained.

@initial   y = output   1971   28 yr = t = x

a. Write an exponential equation to model the decline in population.

$y = ab^x$

$\frac{335}{1170} = \frac{1170}{1170} b^{28}$

$\log 335/1170 = \log b^{28}$

$\frac{\log 335/1170}{28} = \frac{28}{28}\log b$

$\frac{\log 335/1170}{28} = \log_{10} b$

$10^{\wedge}\frac{\log 335/1170}{28} = b$

$10^{-.0194} = b$

$.96 \approx b$

$\underline{y = 1170(.96^t)}$

b. In what year might only 5 bighorn sheep remain in the United States?

$y = 1170(.96^t)$

$\frac{5}{1170} = \frac{1170}{1170}(.96^t)$

$\log\frac{5}{1170} = \log .96^t$

$\frac{\log 5/1170}{\log .96} = t\frac{\log .96}{}$

$\frac{\log 5/1170}{\log .96} = t$

$n=4$

Example 3... An investment of $2,000 earns 5.75% interest, which is compounded quarterly.

a. Write an exponential equation to represent the amount of money in the bank over time.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 2000\left(1+\frac{.0575}{4}\right)^{4t}$$

$$A(t) = 2000(1.014375)^{4t}$$

b. After approximately how many years will the investment be worth $3,000?

$$\frac{3000}{2000} = \frac{2000}{2000}(1.014375)^{4t}$$

$$\log\frac{3}{2} = 4t \log 1.014375$$

$t \approx 7$ yrs

$$\frac{\log 3/2}{4\log 1.014375} = \frac{4t \log 1.014375}{4 \log 1.014375}$$

Example 4... A parent raises a child's allowance by 20% each year. The child's allowance now is $8.

a. Write an exponential equation to represent the child's allowance over time.

$y = ab^x$

$A(t) = 8(1.20)^t$ R

$b = (1 + .20)$

b. When will the child's allowance reach $20?

$$20 = 8(1.20)^t$$

$$\log\frac{20}{8} = t\log 1.2$$

$$\frac{\log 20/8}{\log 1.2} = \frac{t \log 1.2}{\log 1.2}$$

$$\frac{\log 20/8}{\log 1.2} = t \approx 5 \text{ yrs}$$

P $A(t)$ $.08$

Example 5... An investment of \$100 is now valued at \$149. 18. The interest rate is 8%, compounded continuously. About how long has the money been invested?

$$A(t) = Pe^{rt}$$

$$\frac{149.18}{100} = \frac{100}{100} e^{.08t}$$

$$\ln 149.18/100 = \ln e^{.08t}$$

$$\frac{\ln 149.18/100}{.08} = \frac{.08t \cancel{\ln e}^{1}}{.08}$$

$$\frac{\ln(149.18/100)}{.08} = t \approx 5 \text{ years}$$

A

Example 6... An investor sold 100 shares of stock valued at \$34.50 per share. The stock was purchased at \$7.25 per share two years ago. Find the rate of continuously compounded interest in this investment. P

$t = 2$

$$A = Pe^{rt}$$

$$\frac{34.50}{7.25} = \frac{7.25}{7.25} e^{r2}$$

$$\ln 34.50/7.25 = \ln e^{2r}$$

$$\frac{\ln(34.50/7.25)}{2} = \frac{2r}{2} \cancel{\ln e}$$

$$.7799 = r$$

$$\underline{78\% = r}$$