Algebra II
Lesson 7.4: Rational Exponents
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When we hear rational exponent, what do we think of? An exponent that is a rational number. In this section we will learn that when we see $a^{1/2}$, we can identify it with $\sqrt{a}$, or that $a^{1/2} = \sqrt{a}$ Really????? Well square both sides what do you get?

$(a^{1/2})^2 = (\sqrt{a})^2$

$a^{\frac{1}{2}\cdot 2} = a$

$a = a$

The general form:

numerator tells the exponent

$$a^{1/n} = \sqrt[n]{a}$$

The *denominator* *of a fractional exponent is equal to the* *index* *of the radical.* OR the root of the radical

$a^{1/2} \rightarrow n=2 = \sqrt{a}$

$a^{1/3} \quad n=3 = \sqrt[3]{a}$

$a^{1/7} \quad n=7 = \sqrt[7]{a}$

Rewrite using a rational exponent:

$x = x^1$

$\sqrt{5}$	$\sqrt[3]{x^6}$	$\sqrt[2]{16}$	$\sqrt[3]{27}$	$\sqrt[4]{c^3}$
$n=2$ $5^{1/2}$	$n=3$ $(x^6)^{1/3} = x^{6\cdot\frac{1}{3}}$ $= x^{6/3} = x^2$	$16^{1/2}$ $=4$	$n=3$ $27^{1/3}$	$n=4$ $(c^3)^{1/4}$ $= c^{3/4}$

Write in radical form:

$x^{2/7}$	$4^{1/2}$	$x^{8/4}$
$n=7$ $\sqrt[7]{x^2}$	$n=2$ $\sqrt{4}$	$n=4$ $\sqrt[4]{x^8}$ $x^{8/4}$ $= x^2$

We now come to a point where we need to look at a list of rules for rational exponents. If you study and understand these rules, you will find many short cuts for solving problems that have rational exponents.

$a^{1/n} = \sqrt[n]{a},$	$27^{1/3} =$
$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ $a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$	$5^{2/3} = \sqrt[3]{5^2} = (\sqrt[3]{5})^2$
$a^m \cdot a^n = a^{m+n}$	$7^{\frac{1}{3}} \cdot 7^{\frac{2}{3}} = 7^{\frac{1}{3}+\frac{2}{3}} = 7^{3/3} = 7$
$(a^m)^n = a^{mn}$ $(\sqrt{5})^4$	$(5^{1/2})^4 = 5^{\frac{1}{2}\cdot 4} = 5^{\frac{4}{2}} = 5^2$
$(ab)^m = a^m b^m$	$(4 \cdot 5)^{1/2} = 4^{1/2} \cdot 5^{1/2} = 2\sqrt{5}$
$\frac{1}{a^{-m}} = a^m$ $\frac{a^{-m}}{1} = \frac{1}{a^m}$ opposite side opposite sign	$9^{-1/2} =$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{\pi^{3/2}}{\pi^{1/2}} = \pi^{3/2-1/2} = \pi^{2/2} = \pi$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{7}\right)^{1/3} = \frac{5^{1/3}}{7^{1/3}}$

- **Simplest form:** We have learned that simplest form for a radical means that we are to have no radicals in the denominator. Simplest form now includes a rule that for expressions with rational exponents, we need to make every exponent a positive number.

- The property rules that we learned in the previous sections still hold true. We can also use the rules of exponents (summarized below), which include this key formatting principal for exponents, so:
  $5^{1/2} \cdot 5^{1/2} = \sqrt{5} \cdot \sqrt{5} = \sqrt{25} = 5,$ but superimposing rules for exponents we get: $5^{1/2} \cdot 5^{1/2} = 5^{1/2+1/2} = 5.$ *Understanding this technique of converting the radicals into exponential fractions can really help speed up the simplification process.*

Simplify:

$64^{1/3}$	$2^{1/2} \cdot 2^{1/2}$	$7^{1/3} \cdot 7^{2/3}$
$(4^3)^{1/3} = 4^{3/3} = 4$	$2^{\frac{1}{2}+\frac{1}{2}} = 2^{\frac{2}{2}} = 2$	$7^{\frac{1}{3}+\frac{2}{3}} = 7^{\frac{3}{3}} = 7$

$5^{1/3} \cdot 25^{1/3}$ $(5 \cdot 25)^{1/3}$ $5 \cdot 5 \cdot 5$ $(5^3)^{1/3}$ $5^{3/3} = \underline{5}$	$32^{3/5}$ $(2^5)^{\frac{3}{5}} = 2^{15/5}$ $2^{\cancel{5} \cdot 3 / \cancel{5}} = 2^3$ $\underline{= 8}$	$(3x^{2/3})^{-3}$ $3^{-3} \; x^{\frac{2}{\cancel{3}}(-\cancel{3})}$ $\frac{3^{-3} \; x^{-2}}{1}$ $\frac{1}{3^3 \; x^2} = \underline{\frac{1}{27x^2}}$
$(x^{2/3} y^{-1/6})^{-12}$ $x^{\frac{2}{3}(-\cancel{12}^{4})} \; y^{\frac{-1}{\cancel{6}_1}(-\cancel{12}^{2})}$ $\frac{x^{-8} \; y^2}{1}$ $= \frac{y^2}{x^8}$	$25^{-3/2}$ $5^{\cancel{2}(\frac{-3}{\cancel{2}})} =$ $\frac{5^{-3}}{} =$ $\frac{1}{5^3} = \underline{\frac{1}{125}}$	$(8x^{15})^{-1/3}$ $= 8^{\frac{-1}{3}} x^{15(\frac{-1}{3})}$ $= 2^{3(\frac{-1}{3})} x^{\frac{\cancel{15}^{5}}{} \cdot \frac{-1}{\cancel{3}}}$ $= 2^{-1} x^{-5}$ $\underline{\underline{\frac{1}{2x^5}}}$

Remember $2^{-1} = \frac{2^{-1}}{1} = \frac{1}{2}$

Cannot have a fraction without a numerator

$\frac{?}{2}$