Algebra II

Lesson 7.3: Binomial Radical Expressions

Mrs. Snow, Instructor

Like radical terms are radical expressions with the same index and the same radicand. Like radicals may be added or subtracted. This is similar to like variables. $2x^3$ *and* $4x^3$ *are like terms but* $2x^4$ *is different.*

Example: $4\sqrt{5}$ *and* $7\sqrt{5}$ are *like radicals; they are both square roots.* The addition/subtraction property (distribution property) of variables may be applied to numbers with like radicals. Hence, like radicals may be added or subtracted: $4\sqrt{5} + 7\sqrt{5} = 11\sqrt{5}$. We have 4 of something and 7 of the same something so, we have a total of 11 of that something!

$\sqrt{4\cdot 8} = \sqrt{4\cdot 4\cdot 2}$ $\qquad$ $\sqrt{8\cdot 10} = \sqrt{2^3\cdot 2\cdot 5}$

Simplify:

$2\sqrt{7} + 3\sqrt{7}$

$5\sqrt{7}$

$\sqrt{50} + 3\sqrt{32} - 5\sqrt{18}$

$\sqrt{2\cdot\cancel{25}} + 3\sqrt{2\cdot\cancel{16}} - 5\sqrt{\cancel{9}\cdot 2}$

$5\sqrt{2} + 3(4)\sqrt{2} - 5(3)\sqrt{2}$

$5\sqrt{2} + 12\sqrt{2} - 15\sqrt{2}$

$17\sqrt{2} - 15\sqrt{2}$

$2\sqrt{2}$

$3\sqrt{20} - \sqrt{45} + 4\sqrt{80}$

$3\sqrt{\cancel{4}\cdot 5} - \sqrt{9\cdot 5} + 4\sqrt{\cancel{16}\cdot 5}$

$3(2)\sqrt{5} - 3\sqrt{5} + 4(4)\sqrt{5}$

$6\sqrt{5} - 3\sqrt{5} + 16\sqrt{5}$

$3\sqrt{5} + 16\sqrt{5}$

$19\sqrt{5}$

$4\sqrt{xy} + 5\sqrt{xy}$

$9\sqrt{xy}$

$14\sqrt[3]{2} + 3\sqrt[3]{4}$

Ans Simplest form

yes cube root

NO, not liketerms

If $14\sqrt[3]{2} + 3\sqrt[3]{2}$

root & radicand AKA stuff

must be same

$\sqrt{50} + 2\sqrt{72} - \sqrt{12}$

$\sqrt{2\cdot\cancel{25}} + 2\sqrt{\cancel{36}\cdot 2} - \sqrt{\cancel{4}\cdot 3}$

$5\sqrt{2} + 2(6)\sqrt{2} - 2\sqrt{3}$

12

$17\sqrt{2} - 2\sqrt{3}$

$\sqrt{72} = \sqrt{9\cdot 8} = \sqrt{9\cdot 4\cdot 2}$

$36\cdot 2$

$3\sqrt[3]{16} - 4\sqrt[3]{54} + \sqrt[3]{128}$

$3\sqrt[3]{8\cdot 2} - 4\sqrt[3]{27\cdot 2} + \sqrt[3]{8\cdot 8\cdot 2}$

$3(2)\sqrt[3]{2} - 4(3)\sqrt[3]{2} + 2(2)\sqrt[3]{2}$

$6\sqrt[3]{2} - 12\sqrt[3]{2} + 4\sqrt[3]{2}$

$-2\sqrt[3]{2}$

$6\cdot 9$

$2\cdot 3\cdot 3\cdot 3$

128

$64\cdot 2$

$8\cdot 8\cdot 2$

$4^3\cdot 2$

16

$4\cdot 4$

$\underbrace{2\cdot 2\cdot 2}\cdot 2$

$2^3\cdot 2$

$8 = 2^3$

Multiplication of radicals that are in the form of a binomial by be done using the FOIL method.

Exampe:

$(2+\sqrt{7})(1+3\sqrt{7})$

$2 + 6\sqrt{7}$

$+1\sqrt{7} + 21$

$2 + 7\sqrt{7} + 21$

$\underline{\underline{23 + 7\sqrt{7}}}$

$21 = 3(7)$

$3\sqrt{7}\sqrt{7} = 3\sqrt{49}$

$(3+\sqrt{2})(4-\sqrt{5})$

$12 - 3\sqrt{5}$

$+4\sqrt{2} - \sqrt{10}$

$\underline{\underline{12 - 3\sqrt{5} + 4\sqrt{2} - \sqrt{10}}}$

$\sqrt{2}\sqrt{5} = \sqrt{2 \cdot 5}$

$(\sqrt{2}-\sqrt{3})^2$

$(\sqrt{2}-\sqrt{3})(\sqrt{2}-\sqrt{3})$

$2 - \sqrt{6}$

$3 - \sqrt{6}$

$\underline{\underline{5 - 2\sqrt{6}}}$

$\sqrt{2} \cdot \sqrt{2} = \sqrt{2 \cdot 2} = \sqrt{4}$

Conjugates: expressions that only differ in the sign of the second terms. $\sqrt{a}+\sqrt{b}$ *and* $\sqrt{a}-\sqrt{b}$ *are conjugates.* When multiplying conjugates, you will find that the radicals will drop out. Notice the application of the Difference of Two Squares:

Example: $(3+\sqrt{7})(3-\sqrt{7})$

$9 - \cancel{3\sqrt{7}}$

$+ \cancel{3\sqrt{7}} - 7$

$9 - 7 = \underline{\underline{2}}$

$x^2 - 4 = x^2 - 2^2$

$(x-2)(x+2)$

$x^2 + 2x$

$-2x - 4$

$x^2 - 4$

When a radical binomial is in the denominator, multiply by the conjugate to rationalize it:

Example:

$\dfrac{2-\sqrt{3}}{4+\sqrt{3}} \times 1$

$\dfrac{(2-\sqrt{3})}{(4+\sqrt{3})} \cdot \dfrac{(4-\sqrt{3})}{(4-\sqrt{3})} =$

$\dfrac{8 - 2\sqrt{3} - 4\sqrt{3} + 3}{16 - \cancel{4\sqrt{3}} + \cancel{4\sqrt{3}} - 3} =$

$\underline{\underline{\dfrac{11 - 6\sqrt{3}}{13}}}$

$\dfrac{4-2\sqrt[3]{6}}{\sqrt[3]{4}\ \sqrt[3]{2^2}} \cdot \dfrac{\sqrt[3]{2}}{\sqrt[3]{2}} = \dfrac{4\sqrt[3]{2} - 2\sqrt[3]{6 \cdot 2}}{\sqrt[3]{2^2 \cdot 2}}$

$= \dfrac{4\sqrt[3]{2} - 2\sqrt[3]{12}}{\sqrt[3]{2^3}} = \dfrac{4\sqrt[3]{2} - 2\sqrt[3]{12}}{2}$

$\dfrac{\cancel{2}(2\sqrt[3]{2} - \sqrt[3]{12})}{\cancel{2}}$

$= \underline{\underline{2\sqrt[3]{2} - \sqrt[3]{12}}}$

$\sqrt[3]{4} = \sqrt[3]{2^2 \cdot 2}$

$= \sqrt[3]{2^2}\ \sqrt[3]{2}$