Algebra II

Lesson 7-1: Roots and Radical Expressions

Mrs. Snow Instructor

In Chapter 7 we are going to study roots and radical expressions. Remember that **roots or radicals** are the inverse (opposite) of applying **exponents or powers**. You can undo a power with a radical and you can undo a radical with a power. For example$2^2 = 4$ and $\sqrt{4} = \sqrt{2^2} = 2$.

The **square root** of a number a is a number y such that $y^2 = a$. Cube roots, 4th roots, etc. may also be treated in a similar fashion. We also have a definition:

$\sqrt[2]{2^2}$

$$if\ a^n = b, then\ a\ is\ the\ \textbf{nth root}\ of\ b.$$

$\sqrt[n]{a^n} = \sqrt[n]{b}$

$a = \sqrt[n]{b}$

Example: $2^2 = 4$, $2\ is\ the\ square\ root\ of\ 4$ $\quad \sqrt{4} = 2$

$2^3 = 8$, $2\ is\ the\ cube\ root\ of\ 8$ $\quad \sqrt[3]{8} = 2$

$2^4 = 16$, $2\ is\ the\ 4th\ root\ of\ 16$ $\quad \sqrt[4]{16} = 2$

$2^5 = 32$, $2\ is\ the\ 5th\ root\ of\ 32$ $\quad \sqrt[5]{32} = 2$

Recall that the square root of 25 is written as $\sqrt{25}$; and in fact has two possible roots ± 5, because $25 = (+5)^2 = (-5)^2$. The **principal** root (the one that is positive) is 5. Now the cube root of 27 is written as $\sqrt[3]{27} = 3$ and only 3^3can be the solution. The fourth root of 16 is written as $\sqrt[4]{16} = \pm 2$ because $16 = (+2)^4 = (-2)^4$. Again the **principal** root is the positive value, 2.

RULES:

Even roots $(\sqrt[2]{x}, \sqrt[4]{x}, \sqrt[6]{x}, \dots)$ **2 solutions (roots): form of $\pm$**

Calculator: the answer shown will only be the principal root.

REMEMBER: the accurate solution will be the **conjugates ,** $\pm$.

Odd roots $(\sqrt[3]{x}, \sqrt[5]{x}, \sqrt[7]{x}, \dots)$ **1 solution (root): either $+$ or $-$**

radican

Why? A positive or negative number multiplied by itself an ***even*** number of times yields a ***positive*** number. A negative number multiplied by itself an ***odd*** number of times yields a ***negative*** number.

CAUTION!

- **If we are only simplifying a radical, the correct answer is the principal root.**
- **When we set about to solve an equation we need to remember to put a $\pm$ in front of our answer!**
- **The textbook refers to real number roots. Here we assume the positive root.**
- **Two real-number roots would be considered the positive and negative roots.**

Some roots should be straight forward to solve by virtue of your knowledge of the multiplication tables. Rewrite the number or variable in terms of the nth root using Algebra I exponent rules.

When you get to a root that you may be uncertain of, use your calculator to check if the radicand (stuff under radical) is a perfect root:

Example: find each real –number root, with no calculator:

$\sqrt{25}$	$\sqrt{-16}$	$\sqrt[3]{-27}$
$=5$	No Real Solut.	$(-3)(-3)(-3) = 27$ $\quad =-3$

Now, using a calculator: type the nth root on the view screen,	$\sqrt[3]{1728} = 12$	**MATH 4: $\sqrt[3]{1728}$ ENTER 12** (ans)
$\sqrt[4]{81}$		**4 MATH 5:$\sqrt[x]{\ }$ ENTER 81 ENTER 3** answer
$\sqrt[3]{-8} = -2$		**3 MATH 5:$\sqrt[x]{\ }$ ENTER -8 ENTER -2**
$\sqrt[6]{729}$		**6 MATH 5:$\sqrt[x]{\ }$ ENTER 729 ENTER 3** answer

When we introduce a variable under the radical we get a rule we must remember to follow:

> **The nth root rule:** ***For any negative real number a,*** $\sqrt[n]{a^n} = |a|$***, when n is even.***
>
> *When you have a variable raised to an even power and under a radical that has an even index, your answer must be in the absolute value form.*

Why??

$\sqrt{(x^3)^2} =$

$\sqrt{x^6} = x^3$

Yes, we can agree this equation is true, but….
substitute -2 in for x and solve

Evaluate the above equation : $for\ x = -2$

$\sqrt{(-2)^6} \stackrel{?}{=} (-2)^3$

$\sqrt{64}$

$8 \neq -8$

this is a false statement

$\sqrt{x^6} = |x^3|$

$|(-2)^3|$

$= 8$

Now evaluate with the absolute value symbol as the rule states

yep! a true statement as we are looking at the principal root

Even though 64 has two square roots, -8 and 8, as stated earlier, the $\sqrt{\ }$ indicates only the positive root.

REMEMBER: an even index (radical) with an even power variable under the radical will be simplified with answer in absolute value form.

Simplify:

$\sqrt{9x^6} = \sqrt{3^2(x^3)^2}$ $= 3\lvert x^3\rvert$	rewrite radicand such that you have perfect squares where possible the exponent of x is 6/even so we use our nth root rule to solve this radical expression
$\sqrt[3]{8b^6} = \sqrt[3]{2^3(b^2)^3}$ $= 2b^2$	rewrite the radicand such that you have the exponent 3 where possible while the exponent of our variable is even the index (cube root) is odd therefore if b is negative then the root must also be negative.

Squareroot needs square exponent.

$\sqrt{4x^2y^4} = \sqrt{2^2x^2(y^2)^2}$ $= 2\lvert x\rvert\lvert y^2\rvert$ necessary?	rewrite the radicand such that you have exponents equal to the index note the even exponents for the variables, remember the nth root rule even root, even exponent, even exp
$\sqrt[3]{\frac{8}{216}} = \frac{\sqrt[3]{2^3}}{\sqrt[3]{6^3}} = \frac{2}{6} = \frac{1}{3}$ Cuberoot <-> Cube exponent	$\sqrt[4]{x^8y^{12}}$ $\sqrt[4]{(x^2)^4(y^3)^4}$ $= x^2\lvert y^3\rvert$ Answr Abs. val. not needed

even root (4^{th})
even exponent (8)
Answer - even exponent so
abs. val. not necessary

x^2 even if x = negative
x^2 = positive Ans
$\sqrt[4]{x^8}$ = positive Ans

Algebra II

Lesson 7.2: Multiplying and Dividing Radical Expressions

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There are two basic square root properties:

1. **Product Property** of Square Roots: the square root of a number is equal to the product of the square roots of the factors

   example: $\sqrt{20} = \sqrt{5 \cdot 4} = \sqrt{5} \cdot \sqrt{4} = 2\sqrt{5}$ ⟶ $\sqrt{ab} = \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

   $\sqrt{5} \cdot \sqrt{20} = \sqrt{5 \cdot 20} = \sqrt{5 \cdot 5 \cdot 4} = 5 \cdot 2 = 10$ $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b} = \sqrt{ab}$

2. **Quotient Property** of Square Roots: the square root of a quotient is equal to the quotient of the square roots of the dividend and the divisor.

   example: $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$ ⟶ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

   $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$ ⟶ $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

   These properties hold for all radicals. NOTE! You must have the same roots to be able to multiply or divide.

Combine the two integers under the same radical sign, multiply or divide as indicated and simplify if possible.

$\sqrt{3} \times \sqrt{12}$ $= \sqrt{3 \cdot 12} = \sqrt{36}$ $= 6$	$\sqrt[4]{8} \times \sqrt[4]{-4}$ $\sqrt[4]{8(-4)} = \sqrt[4]{-32}$ Not possible	$\frac{\sqrt{12x^4}}{\sqrt{3x}}$ $= \sqrt{\frac{12x^{3}}{3}} = \sqrt{4x^3}$ $= \sqrt{2^2 x^2 x} = 2x\sqrt{x}$
$\frac{\sqrt{24}}{\sqrt{6}} = \sqrt{\frac{24}{6}}$ $= \sqrt{4}$ $= 2$	$\sqrt{2}\sqrt[3]{4}$ cannot simplify	$3\sqrt{7x^3} \cdot 2\sqrt{21x^3y^2}$ $6\sqrt{7 \cdot 21\, x^3 x^3 y^2} =$ $6\sqrt{\cancel{7 \cdot 7} \cdot 3\, \cancel{x^6}\, \cancel{y^2}} =$ $6 \cdot 7\, x^3 y \sqrt{3} = 42x^3y\sqrt{3}$

So here is where the "game of ping pong" comes in. From Lesson 7.1 we learned that for even exponents and even roots we apply an absolute value to the simplified variables. **UNLESS OTHERWISE STATED**, the variables are assumed to be positive and thus we do not need the absolute value symbols in our answers.

You must read directions! This is where you will be told to assume positive variables or to use the absolute value symbols as needed! ***(Live on the edge! Read!)***

Example: Simplify assuming positive variables: Factor into perfect squares; pull out pairs, leaving orphans behind.

$\sqrt{72x^3y^4}$ $9 \cdot 8 \cdot 4 \cdot 2$ $y^2y^2 = y^{2+2}$ $(y^2)^2$ $\sqrt{9 \cdot 4 \cdot 2\, x^2 x}$ 36 $6x\sqrt{2x}$	$\sqrt[4]{64x^3y^6}$ $8 \cdot 8$ $4 \cdot 2\ 4 \cdot 2$ $2^4 \cdot 2^2$ $\sqrt[4]{2^4 2^2 x^3 y^4 y^2}$ $= 2y\sqrt[4]{4x^3y^2}$

Rationalizing the denominator:

When we find a radical in the denominator, it needs to be removed. It is considered improper format and it will be easier to calculate the decimal approximation. The process that clears out radicals in the denominator is called **rationalizing the denominator**. To simply, multiply the fraction by another fraction in the form of "1". Remember that the identity value "1" comes in many forms: $\frac{2}{2}, \frac{-6}{-6}, \frac{\sqrt{3}}{\sqrt{3}}, and\ so\ on.$ The form of 1 which you need to use will be such that the denominator becomes a perfect square, cube, etc. of the radical denominator.

Example: Rationalize: $\frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{3 \cdot 5}}{\sqrt{5 \cdot 5}} = \frac{\sqrt{15}}{5}$

Now rationalize the denominator:

$\sqrt{\frac{7}{3}} = \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$

$\frac{\sqrt{2x^3}}{\sqrt{10xy}} = \sqrt{\frac{2x^3}{10xy}}$

① Simplify

$\sqrt{\frac{x^2}{5y} \cdot \frac{5y}{5y}} = \frac{x\sqrt{5y}}{5y}$

$\frac{4}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$

$\frac{4\sqrt[3]{4}}{\sqrt[3]{2^3}}$

$= \frac{4\sqrt[3]{4}}{2}$

$\frac{\sqrt{5x^{\cancel{4}}\cancel{y}}}{\sqrt{2\cancel{x^2}y^3}}$

×1

$\sqrt{\frac{5x^2}{2y^2}} = \frac{x\sqrt{5}}{y\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{10}}{2y}$

$\frac{3-\sqrt{2}}{2-\sqrt{2}}$