Algebra II
Lesson 6.5/6.6 Finding Roots or Zeros of Cubic Functions
Part II
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Now that we can predict the number and type of roots of a polynomial function we are going to learn how find the roots of a polynomial function. The **Rational Root Theorem** is a technique that identifies all possible rational roots of a polynomial.

How?
Factor both the constant term and the leading coefficient and then make ratios of all possible combinations of the factors**:** $\frac{\boldsymbol{factor\ of\ constant\ term}}{\boldsymbol{factor\ of\ leading\ coefficient}} = \frac{\boldsymbol{p}}{\boldsymbol{q}}$**.** These values are the *possible rational roots.*

Which of these rational roots are actual roots?
When we put the ratios into the polynomial, those that result in a zero (make a true statement) ARE ROOTS.

Take a look at $P(x) = (x-2)(x-3)(x+4)$ multiplying the factors together we get:
$$P(x) = x^3 - x^2 - 14x + 24$$

So the zeros of $P\ are\ 2, 3,\ and -4$. These roots are some of possible rational roots derived using the Rational Root theorem.

Rational Zeros Theorem

If the polynomial, P, has integer coefficients,
then every rational zero of P is of the form $\pm\frac{\boldsymbol{p}}{\boldsymbol{q}}$

p is a factor of the constant coefficient
q is a factor of the leading coefficient.

a. *So you need to find all the possible* $\pm p\ values\ and\ \pm q\ values\ to\ make\ all\ the \pm\frac{p}{q}\ ratios$
   One or more of the $\pm\frac{p}{q}$ *ratios will be zeros of the polynomial.*
   Determine zeros: $P\left(\frac{p}{q}\right) = 0$ *. (is the value of p/q a solution to the polynomial?)*
b. *Once you find a zero, use synthetic division to reduce the polynomial into factors.*
c. *Keep following this process until you reach a quadratic factor then factor the quadratic or use the Quadratic Formula to calculate last two factors.*

$x^3 + x^2 - 3x - 3 = 0$	Identify the constant term and factor: $\pm \boldsymbol{p}$ Identify the leading coefficient and factor : $\pm \boldsymbol{q}$ List all ratio combinations $\pm \frac{p}{q}$ Plug ratios into the polynomial equation and evaluate So, the only rational root is________ *Note: a cubic as 3 solutions the other 2 will be imaginary or irrational.*
$2x^3 + 2x^2 - 19x + 20 = 0$	$2x^3 - 9x^2 - 11x + 8 = 0$

Note: use "store function" key to evaluate expressions for $F\left(\pm \frac{p}{q}\right)$

Find all possible roots and zeros of each cubic polynomial:

1. Using the Rational Root Theorem, find the possible rational roots, 2. If a graphing calculator is available, use the table of values to determine a rational root. 3. Use synthetic division and the rational root to reduce the polynomial, to a linear and quadratic factor. 4. Use the quadratic formula to find the remaining roots. Always check the graph to make sure the roots match the graph.	$x^3 + x^2 - x + 2 = 0$
$2x^3 - x^2 + 2x - 1 = 0$	$y = 2x^3 + 14x^2 + 13x + 6$