Name________________________________

Algebra II

Lesson 6-8

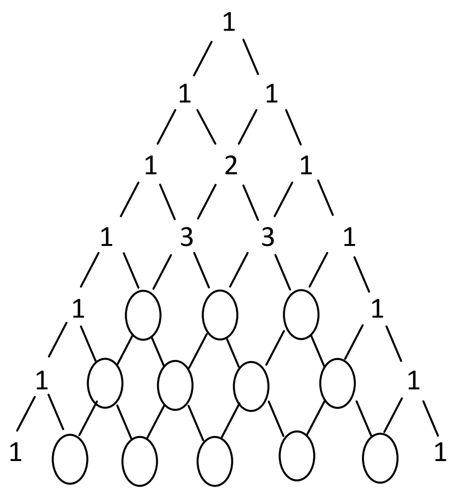
Pascal's Triangle

Suppose we are given a binomial and are told to square it: $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$, no big deal. Now, what if the binomial were to the 3rd power? Harder, but still doable.

$(a+b)^3 = (a+b)(a+b)(a+b) = a^3 + 3a^2b + 3ab^2 + b^3$

What about $(a+b)^6$? That would really be a challenge! With the technique presented in today's lesson, we should find this binomial expansion to be in fact quite straight forward. This technique is called **Pascal's Triangle**. It is named for a French mathematician, Blaise Pascal, who lived during the seventeenth century. However, the earliest known version was developed by an Indian mathematician Halayudha between 300-200 BC.

To make Pascal's Triangle, follow the process below:



$(a+b)^0$;

$(a+b)^1$;

$(a+b)^2$;

$(a+b)^3$;

$(a+b)^4$;

$(a+b)^5$;

$(a+b)^6$

1. Start with a pyramid of three 1's.
2. The second row is created by adding the pairs of numbers from above. Where there is only one number above, you carry down the 1.
3. Each consecutive row is created by adding pairs of numbers from the previous row
4. Now complete row 4,5, and 6;

Now let's look back at our first two expansions: $(a+b)^2$ has the coefficients of 1 – 2 – 1 and $(a+b)^3$ has coefficients 1 – 3 – 3 – 1. These match the numbers in the rows designated for their expansion. So, we can say that the numbers produced by Pascal's Triangle are the coefficients for each binomial expansion. What about the exponents of the expanded binomials? Look back at the expansions we did by hand. **The exponents of each term add up to equal the exponent of our binomial.**

Pascal's Triangle is an expansion of the coefficients of the binomial$(a+b)^n$; the exponents add up to *n.* The first term *a* starts with the exponent *n* and decreases to 0 with every term while *b* starts with 0 and increases to *n*. Let's look at $(a+b)^6$. The coefficients are: 1 – 6 – 15 – 20 – 15 – 6 – 1. Our ab terms are: a^6b^0, a^5b^1, a^4b^2, a^3b^3, a^2b^4, a^1b^5, a^0b^6. Putting it all together:

$$(a+b)^6 = 1a^6b^0 + 6a^5b^1 + 15\,a^4b^2 + 20a^3b^3 + 15\,a^2b^4 + 6a^1b^5 + 1a^0b^6$$

Example: Use Pascal's Triangle to expand $(x-2)^4$

$1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0b^4$

$1x^4(-2)^0 + 4x^3(-2)^1 + 6x^2(-2)^2 + 4x^1(-2)^3 + 1x^0(-2)^4$

$\boldsymbol{x^4 - 8x^3 + 24x^2 - 32x + 16}$

therefore: $(x-2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16$

1. Write out the pattern from Pascal's Triangle
2. Identify $a = x\ and\ b = -2$ and substitute x for a and -2 for b.
3. Simplify