# Algebra 2

## Lesson 6-2 Polynomials and Linear Factors

Just as we factored quadratic equations in Chapter 5, we can factor polynomials with higher degrees. When a polynomial is factored, the terms are known as **Linear Factors.** In math we liken these linear factors to the prime factors of a real number because the polynomial cannoy be factored into any simpler term:

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|---|
| The polynomial $x^3 + 4x^2 + 5x + 2$ in factored form is: $(x+1)(x+1)(x+2)$ |

When a polynomial is in factored form, the **zero product property** may be used to find the zeros. Remember the values of the x-intercepts are called **zeros** because the value of the function is zero at each x-intercept.

**Multiplicity**: If a linear factor of a polynomial is repeated, then the zero is repeated. A **repeated zero** is called a **multiple zero** and has a **multiplicity** equal to the number of times the zero occurs. The exponent of a binomial would indicate the multiplicity

Find the zeros of each function, state the multiplicity

| | |
|---|---|
| twice $(x-3)^2(x-1) = 0$<br>$x-3=0$ $x-3=0$ $x-1=0$<br>$\underbrace{x=3 \quad x=3}_{\text{twice}}$ $x=1$ one time<br>$x=3$ $x=1$<br>multiplicity = 2 mult. = 1 | $(x+1)(x-2)(x-3)$<br>one time each<br>$x+1=0$ $x-2=0$ $x-3=0$<br>$x=1$ $x=2$ $x=3$<br>multiplicity each is one. |

Write a polynomial function given the following zeros

| | |
|---|---|
| $x = -2, 0, 1$<br>$x=-2$ $x=0$ $x=1$<br>$x+2=0$ $x=0$ $x-1=0$<br>$(x)(x+2)(x-1)=0$<br>$(x)(x^2+x-2)=0$<br>$\Rightarrow x^3+x^2-2x=0$ | $x = -5, -5, 1$<br>$x=-5$ $x=-5$ $x=1$<br>$x+5=0$ $x+5=0$ $x-1=0$<br>$(x-1)(x+5)(x+5)=0$<br>$(x-1)(x^2+10x+25)=0$<br>$x^3+10x^2+25x$<br>$-x^2 \quad -10x-25$<br>$\Rightarrow x^3+9x^2+15x-25=0$ |

Factor each polynomial completely

$$9x^3 + 6x^2 - 3x$$

GFC = $3x$

$3x(3x^2 + 2x - 1) = 0$

/ factors

$3x(3x - 1)(x + 1) = 0$

(−1, +1 only factors for −1)

If solving:

$3x = 0$ $\quad$ $3x - 1 = 0$ $\quad$ $x + 1 = 0$

$x = 0$ $\quad$ $x = 1/3$ $\quad$ $x = -1$

$$x^3 + 8x^2 + 16x$$

$x(x^2 + 8x + 16)$

$x(x + 4)(x + 4)$

(note multiplicity of $(x+4)$ factor)

Find the zeros and sketch the graph

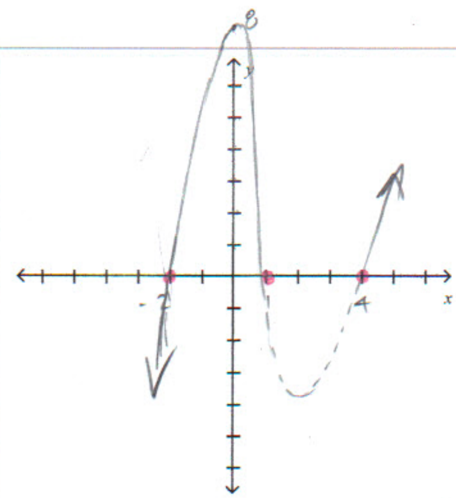
$\cancel{y} = (x - 1)(x + 2)(x - 4) = 0$

$x - 1 = 0$ $\quad$ $x + 2 = 0$ $\quad$ $x - 4 = 0$

$x = 1$ $\quad$ $x = -2$ $\quad$ $x = 4$

multiply out, $x^3$ LC is + cubic function

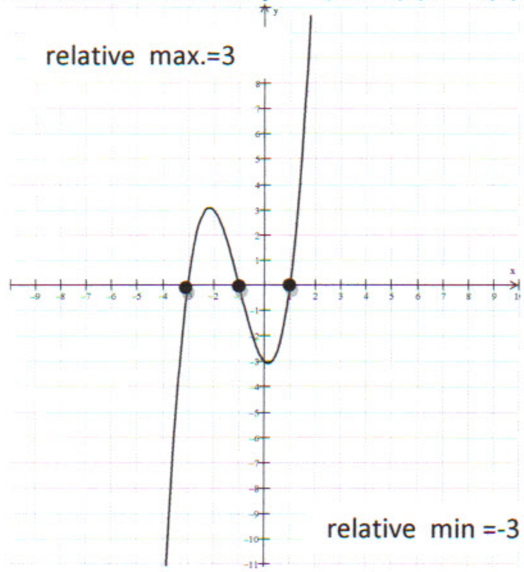
Constants $(-1)(2)(-4) = +8$ y-incpt




we know general shape of $x^3$

With polynomials of degree greater than 2 we may have both minimum and maximum values of **y**. These are called **relative minimum** and **relative maximums** when comparing near-by points on a graph.

**Example:** Find the zeros and relative maximum and relative minimum of: $y = (x+1)(x-1)(x+3)$




1. Using your graphing calculator, enter the equation **Y=**
 note: you don't need to write the expression in polynomial form, enter the binomials using parentheses to separate.

2. What are the relative minimum and maximum?

3. What are the zeros?

**FACTOR THEOREM**: The expression $x - a$ is a linear factor of a polynomial if and only if the value **a** is a zero of the related polynomial factors. In other words: when $x - a$ is a factor,

1. **a** is a solution to the polynomial
2. **a** is an x-intercept of the graph
3. **a** is a zero of the polynomial

## CALCULATOR DIRECTIONS FOR FINDING A CUBIC MODEL

**Example:** Find the cubic model for the following points. (-2,7), (-1,0), (0,1) , (1,2), (2,9)

- First understand that a model is the equation that may be used to "*model*" the data.
- As with data in ch. 5, enter the data in **STAT >Edit 1: edit enter (**L1 is x and L2 is y)
- **2$^{nd}$ --Y=** (stat plot), **1:** turn on the first stat plot
- **ZOOM 9: stat** the data points will be plotted. Here for this data it resembles a possible cubic equation.
- **STAT CALC 6:Cubic Reg** now here before we hit enter twice and got the variables. *Short cut:*
- **VARS > Y-VARS1: Function, 1: Y1** Vars is variables. Hit the VARS key, arrow over to y-variables and arrow down to select function then select Y1
- Here the view screen shows CubicReg Y1. This means we are going to perform the cubic regression and it will be recognized as Y1. **ENTER**
- For this example we get: a=-.166666666667<br>b=2.3333333333<br>c=1.1666666667<br>d=-1.33333333
- Now hit **Y=** and your equation is already entered into Y1. **GRAPH** and the line is graphed as seen in the view screen.