Algebra II
Lesson 6.5/6.6 Finding Roots or Zeros of Cubic Functions
Part II
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Now that we can predict the number and type of roots of a polynomial function we are going to learn how find the roots of a polynomial function. The **Rational Root Theorem** is a technique that identifies all possible rational roots of a polynomial.

How?
Factor both the constant term and the leading coefficient and then make ratios of all possible combinations of the factors: $\frac{\textit{factor of constant term}}{\textit{factor of leading coefficient}} = \frac{p}{q}$. These values are the *possible rational roots.*

Which of these rational roots are actual roots?
When we put the ratios into the polynomial, those that result in a zero (make a true statement) ARE ROOTS.

Take a look at $P(x) = (x-2)(x-3)(x+4)$ multiplying the factors together we get:

$$P(x) = x^3 - x^2 - 14x + 24$$

So the zeros of $P\ are\ 2, 3,\ and -4$. These roots are some of possible rational roots derived using the Rational Root theorem.

Rational Zeros Theorem

If the polynomial, P, has integer coefficients,
then every rational zero of P is of the form $\pm\frac{p}{q}$
p is a factor of the constant coefficient
q is a factor of the leading coefficient.

a. *So you need to find all the possible*
   $\pm p$ *values and* $\pm q$ *values to make all the* $\pm\frac{p}{q}$ *ratios*
   One or more of the $\pm\frac{p}{q}$ *ratios will be zeros of the polynomial.*
   Determine zeros: $P\left(\frac{p}{q}\right) = 0$. *(is the value of p/q a solution to the polynomial?)*
b. *Once you find a zero, use synthetic division to reduce the polynomial into factors.*
c. *Keep following this process until you reach a quadratic factor then factor the quadratic or use the Quadratic Formula to calculate last two factors.*

$$x^3 + x^2 - 3x - 3 = 0$$

$-3 \rightarrow \pm p = \pm 1, \pm 3$

$1 \rightarrow \pm q = \pm 1 \qquad \pm \frac{p}{q} = \pm 1, \pm 3$

$P(1) = 1^3 + 1^2 - 3(1) - 3 \neq 0$

$P(-1) = (-1)^3 + (-1)^2 - 3(-1) - 3 = 0$

$P(3) = 3^3 + 3^2 - 3(3) - 3 \neq 0$

$P(-3) = (-3)^3 + (-3)^2 - 3(-3) - 3 \neq 0$

rational root: -1

Identify the constant term and factor: $\pm \boldsymbol{p}$
Identify the leading coefficient and factor : $\pm \boldsymbol{q}$

List all ratio combinations $\pm \frac{p}{q}$
Plug ratios into the polynomial equation and evaluate

So, the only rational root is -1

Note: a cubic as 3 solutions the other 2 will be imaginary or irrational.

$$2x^3 + 2x^2 - 19x + 20 = 0$$

$20 \rightarrow \pm p = \pm \{1, 2, 4, 5, 10, 20\}$

$2 \rightarrow \pm q = \pm 1, \pm 2$

$\pm \frac{p}{q} = \pm \{1, 2, 4, 5, 10, 20, \frac{1}{2}, \frac{5}{2}\}$

*

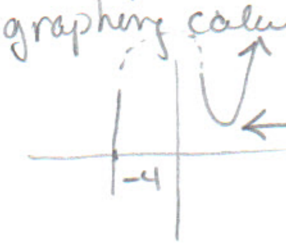
$P(1) = 5 \quad P(2) = 6 \quad P(4) = 104$

$P(-1) = 39 \quad P(-2) = 50 \quad P(-4) = 0$

$P(5) = 225$

$P(-5) = -85$ If you continue you will find no more zeros

graphing calculator shows:



hovering: imaginary solution

$$2x^3 - 9x^2 - 11x + 8 = 0$$

$8 \rightarrow \pm p = \pm \{1, 2, 4, 8\}$

$2 \rightarrow \pm q = \pm \{1, 2\}$

$\pm \frac{p}{q} = \pm \{1, 2, 4, 8, \frac{1}{2}\}$

$P(1) = -10 \quad P(2) = -34 \quad P(4) = -52$

$P(-1) = 8 \quad P(-2) = -22 \quad P(-4) = -220$

$P(8) = 368 \quad P(.5) = .5$

$P(-8) = -1504 \quad P(-.5) = 1.41$

No rational roots

* *Note: use "store function" key to evaluate expressions for* $F\left(\pm \frac{p}{q}\right)$

-4 $\boxed{\text{sto}}$ $\rightarrow$ x enter -4

Find all possible roots and zeros of each cubic polynomial:

1. Using the Rational Root Theorem, find the possible rational roots,
2. If a graphing calculator is available, use the table of values to determine a rational root.
3. Use synthetic division and the rational root to reduce the polynomial, to a linear and quadratic factor.
4. Use the quadratic formula to find the remaining roots.

Always check the graph to make sure the roots match the graph.

Also!! If a calculator is available, put equation into y=, table of values will show x & f(x)

$\Delta x = 1$ on table set will not show fractional answers.

$$x^3 + x^2 - x + 2 = 0$$

$2 \to \pm p = \pm 1, \pm 2$

$1 \to \pm q = \pm 1$ $\quad \pm\frac{p}{q} = \pm 1, \pm 2$

$P(1) = 3$ $\quad P(2) = 12$

$P(-1) = 3$ $\quad P(-2) = 0$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -1 & 2 \\ & & -2 & 2 & -2 \\ \hline & 1 & -1 & 1 & \boxed{0} \end{array}$$

$(x+2)(x^2 - x + 1) = x^3 + x^2 - x + 2$

$a = 1 \quad b = -1 \quad c = 1$

QF: $x = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{2}$

$= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm i\sqrt{3}}{2} = x$

$x = -2$

$$2x^3 - x^2 + 2x - 1 = 0$$

$-1 \to \pm p = \pm 1$

$2 \to \pm q = \pm 1 \pm 2$ $\quad \pm\frac{p}{q} = \pm 1, \pm\frac{1}{2}$

table shows $P(.5) = 0$

so:

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & 2 & -1 \\ & & 1 & 0 & 1 \\ \hline & 2 & 0 & 2 & \boxed{0} \end{array}$$

$(x + \frac{1}{2})(2x^2 + 2) = 2x^3 - x^2 + 2x - 1$

$2x^2 + 2 = 0$

$2x^2 = -2$

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$

$x = \frac{1}{2}$

$$y = 2x^3 + 14x^2 + 13x + 6 = 0$$

solve

$6 \to \pm p = \pm\{1, 2, 3, 6\}$

$2 \to \pm q = \pm 1, \pm 2$

$\pm\frac{p}{q} = \pm\{1, 2, 3, 6, \frac{1}{2}, \frac{3}{2}\}$

table shows $P(-6) = 0$

$$\begin{array}{r|rrrr} -6 & 2 & 14 & 13 & 6 \\ & & -12 & -12 & -6 \\ \hline & 2 & 2 & 1 & \boxed{0} \end{array}$$

$(x+6)(2x^2 + 2x + 1) = 0$

$a = 2 \quad b = 2 \quad c = 1$

$x = \frac{-2 \pm \sqrt{4 - 4(2)(1)}}{4}$

$= \frac{-2 \pm \sqrt{-4}}{4} = \frac{\cancel{-2}^{\,-1} \pm \cancel{2}^{\,1} i}{\cancel{4}_{\,2}}$

$x = \frac{1 \pm i}{2}, 6$