Algebra II
Lesson 6.5/6.6 Theorems about Polynomial Functions
Part I
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The sections of 6.5 and 6.6 are being combined into a single topic with two parts. Part I will cover theorems that will help us to determine all possible roots when only some of the roots are known.

Irrational Root Theorem: Given a polynomial with rational coefficients and $\sqrt{b}$ is irrational. If of $a + \sqrt{b}$ is a root, then you will also have the root of $a - \sqrt{b}$. ; these are called ***conjugates***

opposite sign on 2nd term.

Imaginary Root Theorem: If the imaginary number of $a + bi$ is a root of a polynomial with real coefficients, then the conjugate, $a - bi$ is also a root. Again note these are ***conjugates.*** These roots are called <u>complex conjugates</u>

Same terms - 2nd term opposite sign

To find these roots, you will reduce the polynomial down to linear factors and a quadraticfactor by dividing the real factors into the original polynomial. When you get a quadratic factor, you will use the Quadratic Formula to solve.

A polynomial equation has the following roots, find two additional roots. Conjugates

$2 - \sqrt{7}$ *and* $3 + 2\sqrt{6}$ $2 + \sqrt{7}$, $3 - 2\sqrt{6}$	$1 + \sqrt{3}$, *and* $-\sqrt{11}$ $1 - \sqrt{3}$, $+\sqrt{11}$
$3 - i$ *and* $2i$ $3 + i$, $-2i$	$12 + 3i$, *and* $4 - i$ $12 - 3i$, $4 + i$

note - in the Quadratic Formula the conjugates are "built in" with the $\pm$.

conjugates

$$x = \frac{-b \pm \boxed{\sqrt{b^2 - 4ac}}}{2a} \leftarrow$$ possible complex/irrational value

Find a 3rd degree polynomial equation with rational coefficients that has the given roots

1 *and* $3i \Rightarrow -3i$

$x = 1 \quad x = 3i \quad x = -3i$

$x-1=0 \quad x-3i=0 \quad x+3i=0$

conjugates

$(x-1)(x-3i)(x+3i)=0$

$(x-1)(x^2-9\cancel{i^2}^{-1})=0$

$(x-1)(x^2+9)=0$

x^3+9x

$-x^2-9$

$x^3-x^2+9x-9=0$

$2+i$ *and* -3

$2-i$

$x=-3 \quad x=2+i \quad x=2-i$

$x+3=0 \quad x-2-i=0 \quad x-2+i$

conjugates

$(x+3)((x-2)-i)((x-2)+i)=0$

$(x+3)[(x-2)(x-2)-\cancel{i^2}^{-1}]=0$

$(x+3)[x^2-4x+\overset{5}{\cancel{4+1}}]=0$

x^3-4x^2+5x

$3x^2 \quad +15$

$x^3-x^2+5x+15=0$

Find a 4th degree polynomial equation with rational coefficients that has the given roots:

$5-i$ and $\sqrt{2} \Rightarrow$ conjugates $5+i$, $-\sqrt{2}$

$x=5-i \quad x=5+i \quad x=\sqrt{2} \quad x=-\sqrt{2}$

$x-5+i=0 \quad x-5-i=0 \quad x-\sqrt{2}=0 \quad x+\sqrt{2}=0$

$((x-5)+i)((x-5)-i)(x-\sqrt{2})(x+\sqrt{2})=0$

$((x-5)^2-\cancel{i^2}(-1))(x^2-2)=0$

$(x^2-10x+\overset{26}{\cancel{25+1}})(x^2-2)=0$

$(x^2-2)(x^2-10x+26)=0$

$x^4-10x^3+26x^2$

$-2x^2+20x-52=0$

$x^4-10x^3+24x^2+20x-52=0$

Fundamental theorem of Algebra

In 1797 Carl Gauss proved what is known as the **Fundamental Theorem of Algebra.** It states that *a polynomial of degree $n \geq 1$ has at least one complex zero. In essence,* the fundamental theorem of algebra guarantees that every polynomial has a complete factorization, if we are allowed to use complex numbers $(a + bi)$. Remember that a real number may be written as a complex number.

Fundamental Theorem of Algebra; *translation!:* An nth degree polynomial equation has exactly n roots; at least one of them will be complex.

You can often find all the zeros of a polynomial function by using a combination of graphing, Factor Theorem, polynomial division, the Remainder Theorem and the Quadratic Formula.

Find the number of complex roots, and the possible number of real roots

$$x^5 + x^3 + 2x^2 - 6 = 0$$

degree 5 = 5 complex roots

imaginary roots will be paired ±i

(a + __)(a − __)(c + __)(c − __)(d)

at most 4 imaginary → 1 real
2 imaginary → 3 real
0 " → 5 real

$$x^{10} + x^8 + x^4 + 3x^2 - x + 1 = 0$$

10 complex roots (degree)
imaginary roots pair up ± i
(a +)(a −)(b +)(b −)(c +)(c −)(d +)(d −)
(e +)(e −)

so: all imaginary (10) – 0 real roots
8 " 2 real
6 " 4 real
4 " 6 real
2 " 8 real
0 " all (10) real