Algebra 2
Lesson 6-4: Solving Polynomial Equations
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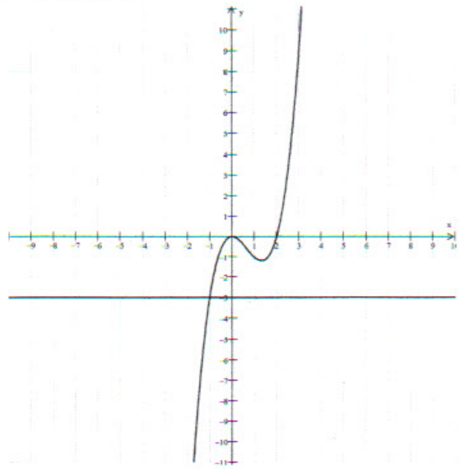
When an equation has a polynomial on each side, you can solve the equation by graphing each side separately and finding the x values at the points of intersection.

Example: Graph and solve: $x^3 - 2x^2 = -3$



1. Graph $y_1 = x^3 - 2x^2$
2. Graph $y_2 = -3$
3. To find the intersection point use: **2nd TRACE – 5:intersect**

Where do the two equations intersect?

This is very similar to what we did in Ch.3; we found the solution of systems of linear equations graphically by finding intersection of two lines

The Sum and Differences in Cubes:

$$\boldsymbol{a^3 + b^3 = (a+b)(a^2 - ab + b^2)}$$
$$\boldsymbol{a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$$

Factor:

$x^3 + 64$	$8x^3 - 1$
$x^3 + 4^3$ $(x+4)(x^2 - 4x + 16)$	$(2x)^3 - 1^3$ $(2x-1)(4x^2 + 2x + 1)$

To solve a polynomial equation: f(x)=0, we can use the same techniques we used in Ch. 5.

$x^3 + 8 = 0$	
$x^3 + 2^3 = 0$ $(x+2)(x^2 + 2x + 4) = 0$ $x + 2 = 0 \quad x^2 + 2x + 4 = 0 \quad a = 1\ b = 2\ c = 4$ $\underline{\underline{x = -2}} \qquad x = \dfrac{-2 \pm \sqrt{4 - 4(1)(4)}}{2(1)}$ $= \dfrac{-2 \pm \sqrt{4 - 16}}{2}$ $= \dfrac{-2 \pm \sqrt{-12}}{2} =$	identify the **Sum of cubes** **Zero-Product Property** use the quadratic formula to solve $\therefore x = -2$ $x = -1 \pm 2\sqrt{3}$ $\dfrac{-\cancel{2}^{1} \pm \cancel{4}^{2} i\sqrt{3}}{\cancel{2}_{1}} = -1 \pm 2\sqrt{3}$

$x^3 - 125 = 0 \rightarrow x^3 - 5^3 = 0$

$(x-5)(x^2+5x+25) = 0 \qquad a=1 \quad b=5 \quad c=25$

$x-5=0 \qquad x^2+5x+25=0$

$x=5 \qquad x = \dfrac{-5 \pm \sqrt{25-4(1)(25)}}{2(1)}$

$= \dfrac{-5 \pm \sqrt{25-100}}{2}$

$= \dfrac{-5 \pm \sqrt{-75}}{2}$

$x=5 \qquad x = \dfrac{-5 \pm 5i\sqrt{3}}{2}$

$8x^3 - 1 = 0 \rightarrow (2x)^3 - 1^3 = 0$

$(2x-1)(4x^2+2x+1) = 0$

$a=4 \quad b=2 \quad c=1$

$2x-1=0$

$x = 1/2$

$x = \dfrac{-2 \pm \sqrt{4-4(4)(1)}}{2(4)}$

$= \dfrac{-2 \pm \sqrt{4-16}}{8}$

$= \dfrac{-2 \pm \sqrt{-12}}{8}$

$= \dfrac{-\cancel{2}^{1} \pm \cancel{2}^{1} i\sqrt{3}}{\cancel{8}\,4}$

$x = 1/2, \quad x = \dfrac{1 \pm i\sqrt{3}}{4}$

When solving higher-degree polynomials, consider using substitution to make factoring easier:

Solve:

$x^4 + 11x^2 + 18 = 0, \qquad let\ u = x^2 \ \therefore u^2 = x^4$ — substitute $u = x^2$ and rewrite the equation

$u^2 + 11u + 18 = 0$

$(u+9)(u+2) = 0$ — factor

$(x^2+9)(x^2+2) = 0$ — now put back into the $x\ form$ and solve for x

$(x^2+9) = 0 \ \ or\ (x^2+2) = 0$

$x^2 = -9 \quad or\ x^2 = -2$

$\boldsymbol{x = \pm 3i \quad or \quad x = \pm i\sqrt{2}}$

Factor and then solve:
$x^4 - 8x^2 + 7 = 0$ (use substitution)

$u = x^2 \rightarrow u^2 = x^4$

$u^2 - 8u + 7 = 0$

$(u-7)(u-1) = 0$

replace with original variable $\therefore$

$(x^2-7)(x^2-1) = 0$

$x^2 - 7 = 0 \qquad x^2 - 1 = 0$

$x^2 = 7 \qquad x^2 = 1$

$x = \pm\sqrt{7} \qquad x = \pm 1$

The width of a box is 2 m less than the length. The height is 1 m less than the length. The volume is 60 m³. Find the length of the box.

as with page 1 example: Set up equations & graph

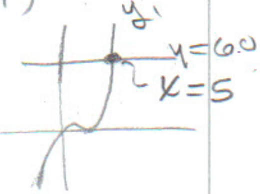
$w = l-2 \quad h = l-1 \quad V = 60m^3$

$l(l-2)(l-1) = 60$

calculator:

$y_1 = l(l-2)(l-1)$

$y_2 = 60$



$l = 5m \Rightarrow w = 3m \ \ h = 4m$