Algebra 2

Lesson 6-3: Dividing Polynomials

Mrs. Snow, Instructor

Dividing two numbers we use a process known as long division.

$1512 \div 4$

$$\begin{array}{r} 378 \\ 4\overline{)1512} \\ -12\downarrow \\ \hline 31 \\ -28\downarrow \\ \hline 32 \\ -32 \\ \hline 0 \end{array}$$

$(378)(4) = 1512$

$1649 \div 7$

OR

$$\begin{array}{r} 235 \text{ r } 4 \\ 7\overline{)1649} \\ 14\downarrow \\ \hline 24 \\ 21 \\ \hline 39 \\ 35 \\ \hline 4 \end{array}$$

4 ← Remainder

We can also polynomials:

$x^2 + 3x - 18 \; by \; x - 3$

$$\begin{array}{r} x + 6 \\ x-3\overline{)x^2 + 3x - 18} \\ -(x^2 - 3x) \quad \downarrow \\ \hline 0 + 6x - 18 \\ -(6x - 18) \\ \hline 0 \end{array}$$

$x(?) = x^2$
$? = x$
$x(?) = 6x$
$? = 6$

$(x-3)(x+6) = x^2+3x-18$

1. look at the first term in each polynomial. Here, ask, *x goes into x^2 how many times?*
2. As with long division, multiply quotient by the divisor.
and simplify; drop the next term from the dividend

3. Repeat the process of bringing down the next term followed by dividing, multiplying, and subtracting

$x^2 + 2x - 30 \div x - 5$

$$\begin{array}{r} x + 7 \\ x-5\overline{)x^2 + 2x - 30} \\ -(x^2 - 5x) \\ \hline 7x - 30 \\ -(7x - 35) \\ \hline 5 \end{array}$$

5 Remainder

$x^2+2x-30 =$

$(x-5)(x+7) + 5$

$x^3 + 7x^2 - 4 \div x + 2$

$$\begin{array}{r} x^2 + 5x - 10 \\ x+2\overline{)x^3 + 7x^2 + 0x - 4} \\ -(x^3 + 2x^2) \\ \hline 5x^2 + 0x \\ -(5x^2 + 10x) \\ \hline -10x - 4 \\ -(-10x - 20) \\ \hline 16 = R \end{array}$$

"0x" place holder

$x^3+7x^2-4 =$

$(x+2)(x^2+5x-10) + 16$

When there is a remainder, the proper form for the factor is:

(dividend) = (divisor)(quotient) + remainder

How does this dividing help us?

1. Given a factor, we can simplify by dividing to find the factor pair.
2. We can verify if a polynomial is a factor of another polynomial. If the remainder is zero then our divisor is a factor!

Remainder Theorem

If we have a polynomial $P(x)$ and it is divided by $x - a$, then:

$$P(a) = number = remainder$$

A second type of division we can use which is quicker than long division is known as **synthetic division**. This technique works only when we have a **linear binomial in the form of** $x - a$, *that is* $x - \boxed{a}$

$x^3 - 7x^2 + 15x - 9 \div x - 3$

a| coefficients

$x - \boxed{3}$

$a = 3$

$$\begin{array}{r|rrrr} 3 & 1 & -7 & 15 & -9 \\ & \downarrow & 3 & -12 & 9 \\ \hline & 1 & -4 & 3 & \boxed{0} \end{array}$$

← Remainder

← coefficient & const of polyn 1 degree less

so

(linear)(Quadratic) = cubic

$(x-3)(1x^2 - 4x + 3) = x^3 - 7x^2 + 15x - 9$

$x^3 + 4x^2 + x - 6 \div x + 1$

$x - \boxed{-1}$

$a = -1$

$$\begin{array}{r|rrrr} -1 & 1 & 4 & 1 & -6 \\ & \downarrow & -1 & -3 & 2 \\ \hline & 1 & 3 & -2 & \boxed{-4} \end{array}$$

Remainder

$(x+1)(x^2 + 3x - 2) + 4$

Is $(x + 2)$ a factor of:

$2x^2 + 7x + 6$

$x - (-2)$

$a = -2$

$$\begin{array}{r|rrr} -2 & 2 & 7 & 6 \\ & \downarrow & -4 & -6 \\ \hline & 2 & 3 & \boxed{0} \end{array}$$

← even no remainder

Yes $(x+2)$ is a factor

$x^3 - 5x - 10$

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & c \\ -2 & 1 & 0 & -5 & -10 \\ & & -2 & 4 & 2 \\ \hline & 1 & -2 & -1 & \boxed{-8} \end{array}$$

Remainder

All terms!

Not fact

The volume in cubic feet of a workshop's storage chest can be expressed as the product of its three dimensions by the given function: $V(t) = x^3 + 7x^2 + 10x$. The depth of the chest is given by the function $(x + 2)$. Find the linear expressions for the other two dimensions.

$a = -2$

	x^3	x^2	x	c
-2	1	7	10	0
		-2	-10	0
	1	5	0	$\boxed{0}$

$V(t) = (x+2)(x^2+5x+0)$

$= x+2)(x^2+5x)$

$= (x+2)\underbrace{(x)(x+5)}_{\text{other dimensions}}$

Find $P(4)$ for $P(x) = x^4 - 5x^2 + 4x + 12$ use synthetic division:

$a = 4$

4	1	0	-5	4	12
		4	16	44	192
	1	4	11	48	$\boxed{204}$

$P(4) = \underline{204}$

Same!

Now solve for $P(+4)$

$P(4) = 4^4 - 5(4^2) + 4(4) + 12$

$= 256 - 80 + 16 + 12$

$= 204$

Find $P(-1)$ for $P(x) = 2x^4 + 6x^3 - 5x^2 + 60$

-1	2	6	-5	0	60
		-2	-4	9	-9
	2	4	-9	9	$\boxed{51}$

$P(-1) = \underline{51}$

$P(-1) = 2(-1^4) + 6(-1^3) - 5(-1^2) + 60$

$= 2 - 6 - 5 + 60$

$= 51$