5.6 Complex Numbers

$\sqrt{4} = \pm 2$

$\sqrt{-4}$ NO Real Solutions

Complex Numbers

- Real Numbers: π, $\sqrt{2}$, $\frac{5}{2}$, $\frac{1}{7}$, $-4, -3, -2$, 0, $1, 2, 3 \ldots$
- Imaginary Numbers: $i = \sqrt{-1}$

↑ i is a number not a variable

By definition $\sqrt{-1} = i$

$-1 = i^2$ Square both sides

Complex Number $\Rightarrow$ $a + bi$

a → Real part; bi ← imaginary part

Simplify: $\sqrt{-4} = \sqrt{-1 \cdot 4} = \sqrt{4} \cdot \sqrt{-1} = \pm 2i$

$\sqrt{-12} = \sqrt{-1 \cdot 4 \cdot 3} = \sqrt{-1} \cdot \sqrt{4}\sqrt{3} = \pm 2i\sqrt{3}$

$\sqrt{-65} = \sqrt{-1}\sqrt{5 \cdot 13} = i\sqrt{65}$

Write as a complex number $a + bi$

$2 \Rightarrow 2 + 0i$ $\quad -3i \Rightarrow 0 - 3i$

$3 + \underbrace{\sqrt{-4}}_{2i} \Rightarrow 3 + 2i$ $\quad 5 - \underbrace{\sqrt{-12}}_{2i\sqrt{3}} \Rightarrow 5 - 2i\sqrt{3}$

add/subtract complex numbers

$(2+3i)+(4-5i)$ add real part, add imaginary part

$= 2+4+3i-5i$

$= 6-2i$

$(4+3i)-(1-2i) = (4+3i)-1+2i$

$= 4-1+3i+2i = 3+5i$

Additive Inverse - a number and its opposite

inverse = 0

$2 \rightarrow$ inverse -2 $\quad 2-2=0$

$3i$ inverse $-3i$ $\quad 3i-3i=0$

Multiply complex numbers

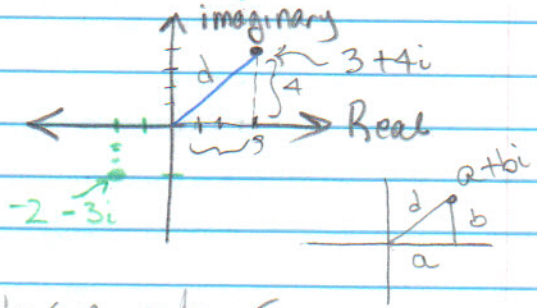
$(3i)(2i) = 3\cdot 2\cdot i\cdot i = 6i^2 = 6(-1) = -6$ ($i^2=-1$)

$(2+3i)(3-4i) = 6-8i+9i-12i^2$ ($i^2=-1$)

$6+i+12 = 18+i$

Graph: $3+4i$

$-2-3i$



Absolute value of a complex number

$|a+bi| = \sqrt{a^2+b^2}$

$|3+4i| = \sqrt{d^2} = \sqrt{3^2+4^2}$

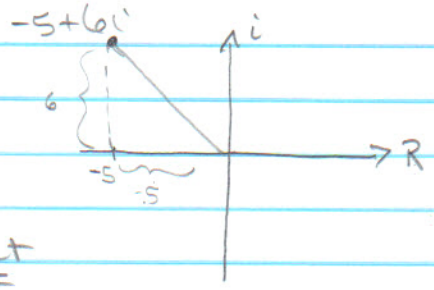
$d = \sqrt{9+16} = \sqrt{25}$

$d = 5$

$|-5+6i| = \sqrt{-5^2+6^2}$

$= \sqrt{25+36}$

$|-5+6i| = \sqrt{61}$ ← exact



Solve by square roots:

$x^2+36=0$

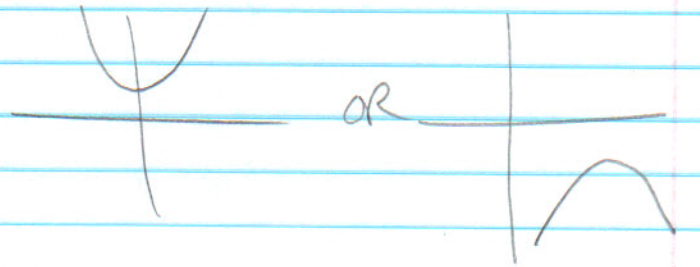
$\sqrt{x^2} = \sqrt{-36}$

$x = \sqrt{-36} = \sqrt{-1}\sqrt{36} = \pm 6i$

$x^2+1=0$

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$



discriminant b^2-4ac for $y=ax^2+bx+c$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$ ← axis of symmetry

$b^2-4ac=0 \Rightarrow$ 1 real solution

$b^2-4ac>0$ 2 real solutions

$b^2-4ac<0$ 2 imaginary solutions

$y = x^2+5x+6$ $\quad b^2-4ac$

$a=1\ b=5\ c=6$ $\quad 25-4(1)(6)$

2 real $\quad 25-24=1$

$y = 2x^2+5x+6$ $\quad 25-4(2)(6)$

2 imaginary sol. $\quad 25-48 \Leftarrow$ neg