Algebra 2

Lesson 5-5: Quadratic Equations

$y = ax^2+bx+c$

Mrs. Snow, Instructor

When solving a quadratic equation: $ax^2 + bx + c = 0$, we are looking for the solutions of x $when$ $y = 0$. There several ways we can solve for **x**. One way is through **factoring:**

In the last section, we learned how to factor a quadratic expression. This skill will enable us to find solutions to x algebraically when we use the **Zero-Product Property.**

Zero-Product Property: $If\ ab = 0, then\ a = 0\ or\ b = 0$. (If a product of 2 values equals zero, it stands to reason that one or the other term will have to be equal to zero)

$(-4+4)(-4+8) = 0$

Example: $(x + 4)(x + 8) = 0, then\ (x + 4) = 0\ or\ (x + 8) = 0$ from here we can solve these 2 little equations for x:

$x + 4 = 0$ $\quad x + 4 - 4 = 0 - 4$ OR $\quad x + 8 = 0$ $\quad x + 8 - 8 = 0 - 8$

$x = -4$ $\quad\quad x = -8$

In the case of a quadratic, both of these x- values are solutions to the equation; they are the points where the parabola will cross the x-axis

Let's put the whole picture together: ARRGH! With a harder problem! (*but good review*)

Example: Solve for x by Factoring:

$ax^2+bx+c=0$

$x^2 - 7x - 18 = 0$

$(-)(+)$ $\quad 1\cdot 18$ $\quad 2\cdot 9$ $\quad 3\cdot 6$ $\quad +2-9=-7$

$(x+2)(x-9)=0$

$x+2=0$ $\quad x-9=0$

$x=-2$ $\quad x=9$

$2x^2 - 4x = 6$

$2x^2-4x-6=0$

$\left(\tfrac{1}{2}\right)2(x^2-2x-3)=0\left(\tfrac{1}{2}\right)$

$x^2-2x-3=0$

$(-)(+)$ $\quad 1\cdot 3$ $\quad 1-3=-2$

$(x+1)(x-3)=0$

$x+1=0$ $\quad x-3=0$

$x=-1$ $\quad x=3$

$3x^2 - 20x - 7 = 0$

$(a)(c)$ $\quad (3)(-7)$ $\quad -21$ $\quad (+)(-)$ $\quad 3\cdot 7$ $\quad (1)(-21)$ $\quad +1-21=-20$

$3x^2+1x-21x-7=0$

$x(3x+1)-7(3x+1)=0$

$(3x+1)(x-7)=0$

$3x+1=0$ $\quad x-7=0$

$\frac{3x}{3}=\frac{-1}{3}$ $\quad x=7$

$x=\frac{-1}{3}$

$3x^2 = -5x + 12$

$(3)(-12)=-36$ $\quad (+)(-)$ $\quad 1\cdot 36$ $\quad 2\cdot 18$ $\quad 3\cdot 12$ $\quad 4\cdot 9$ $\quad 6\cdot 6$ $\quad +9-4$

$3x^2+5x-12=0$

$3x^2+9x-4x-12=0$

$3x(x+3)-4(x+3)=0$

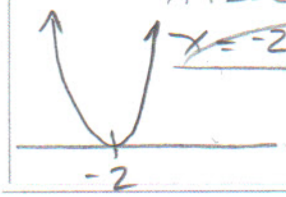
$(x+3)(3x-4)=0$

$x+3=0$ $\quad 3x-4=0$

$x=-3$ $\quad \frac{3x}{3}=\frac{4}{3}$

$x=\frac{4}{3}$

$\frac{1}{3}(3x^2 + 12x + 12) = 0(\frac{1}{3})$ $x^2+4x+4=0$ $(x+2)(x+2)=0$ $(+)(+)$ $1\cdot 4$ $2\cdot 2$ $x+2=0$ $x+2=0$ $x=-2$ $x=-2$	a^2-b^2 $(a+b)(a-b)$ $x^2 - 64 = 0$ $x^2-8^2=0$ $x^2+0x-64$ $(+)(-)$ $(x-8)(x+8)=0$ $(-8)(8)$ $x-8=0$ $x+8=0$ $x=8$ $x=-8$

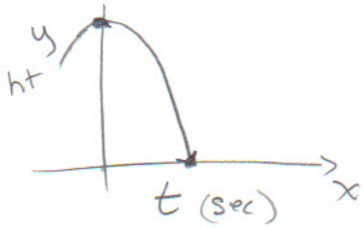


Yes, there are some problems that are so simple you may wonder.

Solve using square roots

$x^2 - 25 = 0$ $+25$ $+25$ $\sqrt{x^2}=\sqrt{25}$ $x=\pm 5$	$3x^2 - 24 = 0$ $+24$ $+24$ $\frac{3x^2}{3}=\frac{24}{3}$ $\sqrt{x^2}=\sqrt{8}$ $x=\pm 2\sqrt{2}$ $\sqrt{2\cdot 4}$ $\sqrt{2}\sqrt{4}=2\sqrt{2}$	$3x^2 + 27 = 0$ -27 -27 $\frac{3x^2}{3}=\frac{-27}{3}$ $\sqrt{x^2}=\sqrt{-9}$ $x=\pm$ No real solutions

The tallest building in the world is the Burj Kalifah in Dubai. It stands 2,722 feet tall. The function, $y = -16t^2 + 2722$ models the height in y in feet of an object t seconds after it is dropped from the top of the building. how long will it take the object to hit the ground?



$-16t^2+2722=0$

$-2722=0$

$\frac{-16t^2}{-16}=\frac{-2722}{-16}$

$\sqrt{t^2}=\sqrt{\frac{2722}{16}}$

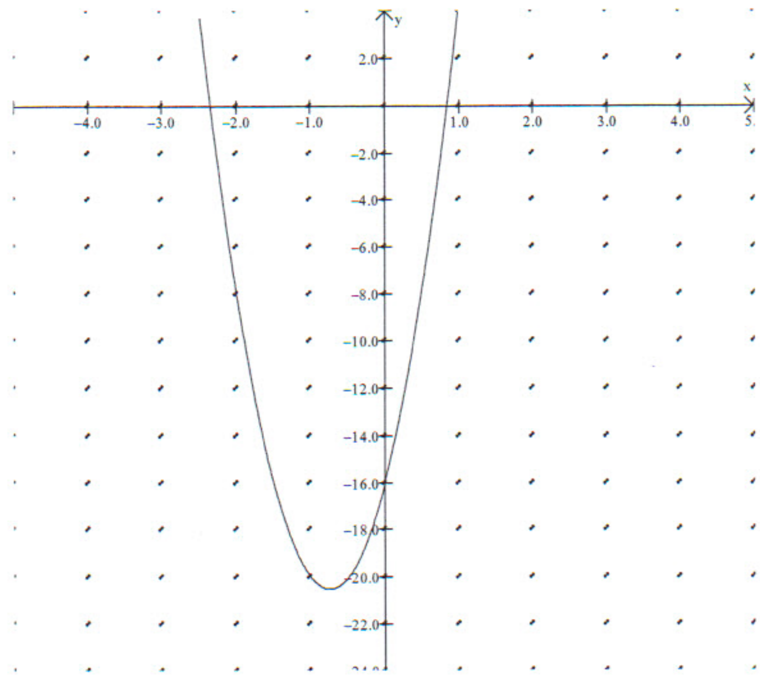
~~$t \approx \pm 13$ seconds~~

$t \approx 13$ sec

GRAPHING

Not every quadratic is factorable. In these cases we can graph the quadratic equation and find the solutions to the equation off the graph.

Example: Using the graphing calculator, graph
$8x^2 + 12x - 16 = 0$



What do you see?
That is, where does the parabola cross the x-axis?
ANS.: At the x-intercepts! These are the points where y is equal to 0 and are called **zeros of the function** or **the roots of the equation**.

In other words, if we graph the parabola on the calculator then, **2nd TRACE, 2: zero, ENTER**, and follow the directions to identify the left and right bounds WRT the parabola intersecting the x-axis, you will get the zeros for the equation. Note: you will need to do this process twice so to find both **zeros of the function.**

$\boldsymbol{x = -2.35 \text{ or } 0.85}$

Algebra 2

Lesson 5-8: The Quadratic Formula

Mrs. Snow, Instructor

So far, you have learned that a quadratic equation can be solved by graphing, factoring, and square rooting. You also can solve for x's that are complex. There is yet another method of factoring called **the Quadratic Formula.** I call it the "Queen Bee," because it is the Queen; it may be used to factor any quadratic equation.

Given a quadratic equation, $ax^2 + bx + c = 0$, the roots or zeros can be found by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x^2 + 3x + 7$

First off….

Simplify the square roots:

$\sqrt{12}$ $\sqrt{4 \cdot 3}$ $\sqrt{4} \cdot \sqrt{3}$ $2\sqrt{3}$	Simplest form $\sqrt{30}$ $\sqrt{5 \cdot 6}$ $\sqrt{5 \cdot 3 \cdot 2}$	$\sqrt{72} = 6\sqrt{2}$ $\sqrt{9 \cdot 8}$ $\sqrt{9 \cdot 4 \cdot 2}$ $\sqrt{9}\sqrt{4}\sqrt{2}$ $3 \cdot 2 \cdot \sqrt{2}$

Solve using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$1x^2 + 4x + 3 = 0$ $a = 1 \quad b = 4 \quad c = 3$ $x = \frac{-4 \pm \sqrt{16 - 4(1)(3)}}{2(1)}$ $= \frac{-4 \pm \sqrt{16 - 12}}{2}$ $= \frac{-4 \pm \sqrt{4}}{2}$ $= \frac{-4 \pm 2}{2}$ $\frac{-4+2}{2} = \frac{-2}{2} = -1$ $\frac{-4-2}{2} = \frac{-6}{2} = -3$ Ans $x = -1, -3$	$x^2 = 6x - 1$ $1x^2 - 6x + 1 = 0 \quad a = 1 \quad b = -6 \quad c = 1$ $x = \frac{6 \pm \sqrt{36 - 4(1)(1)}}{2(1)}$ $= \frac{6 \pm \sqrt{36 - 4}}{2}$ $= \frac{6 \pm \sqrt{32}}{2}$ $= \frac{6 \pm 4\sqrt{2}}{2} = \frac{\cancel{2}(3 \pm 2\sqrt{2})}{(\cancel{2})} = 3 \pm 2\sqrt{2}$ Ans $\sqrt{32}$ $\sqrt{8 \cdot 4}$ $\sqrt{2 \cdot 4 \cdot 4}$ $\sqrt{2}\sqrt{16}$ $4\sqrt{2}$
$2x^2 + 7x + 5 = 0$ $a = 2 \quad b = 7 \quad c = 5$ $x = \frac{-7 \pm \sqrt{49 - 4(2)(5)}}{2(2)}$ $= \frac{-7 \pm \sqrt{49 - 40}}{4}$ $= \frac{-7 \pm \sqrt{9}}{4}$ $= \frac{-7 \pm 3}{4}$ $\frac{-7+3}{4} = \frac{-4}{4} = -1$ $\frac{-7-3}{4} = \frac{-10}{4} = -\frac{5}{2}$ Ans $x = -\frac{5}{2}, -1$	$x^2 + 9x - 18 = 0$ $a = 1 \quad b = 9 \quad c = -18$ $x = \frac{-9 \pm \sqrt{81 - 4(1)(-18)}}{2(1)}$ $= \frac{-9 \pm \sqrt{81 + 72}}{2}$ $= \frac{-9 \pm \sqrt{153}}{2} = \frac{-9 \pm 3\sqrt{17}}{2}$ Ans $\sqrt{153} =$ $= \sqrt{9 \cdot 17}$ $= \sqrt{9}\sqrt{17}$ $3\sqrt{17}$

The **discriminant** of a quadratic equation is $\boldsymbol{b^2 - 4ac}$. This expressions will help your to determine **how many** and **what kind of roots** a quadratic equation will have.

- If $b^2 - 4ac > 0$, then the quadratic equation will have **TWO** real roots.
- If $b^2 - 4ac = 0$, then the quadratic equation will have **ONE** real root.
- If $b^2 - 4ac < 0$, then the quadratic equation will have **NO** real roots.

How many and what kind of roots do the quadratic equations have? b^2-4ac

$y = 2x^2 + x + 28$	$2x^2 + 7x - 15 = y$	$x^2 - 12x + 36 = y$
$a=2$ $b=1$ $c=28$	$a=2$ $b=7$ $c=-15$	$a=1$ $b=-12$ $c=36$
$1^2-4(2)(28)$	$49-4(2)(-15)$	$144-4(1)(36)$
$1-224$	$49+120$	$144-144$
-223	$=169>0$	$=0=0$
$b^2-4ac<0$ $\therefore$ NO real sol.	$\therefore$ 2 real solutions	$\therefore$ 1 real solution

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \xrightarrow{0}$$

$$\frac{-b}{2a} \rightarrow \text{1 solution real}$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \text{positive \#}$$

$$\frac{-b+\#}{2a}, \quad \frac{b-\#}{2a} \quad \text{2 real solutions}$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \sqrt{neg}$$

$$= \frac{-b \pm \sqrt{-\#}}{2a} \quad \text{not real} :)$$