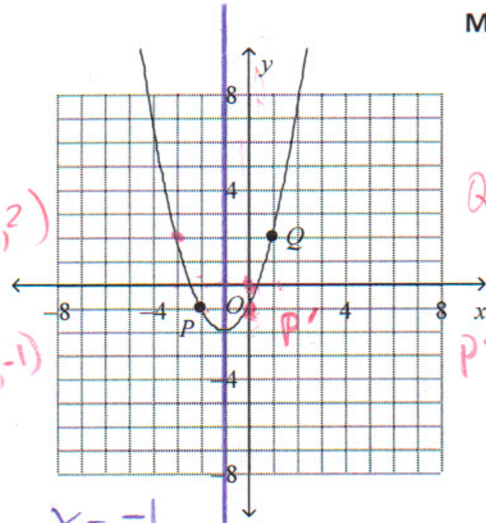
Algebra 2
Lesson 5-1: Modeling Data with Quadratic Functions
Mrs. Snow, Instructor



Q' (-3, 2)

Q = (1, 2)

P = (-2, -1)

P'

P' (0, -1)

x = -1

x = real #

axis of symmetry

Is the graph to the left a function? How so? Passes vertical line test

What is the name of the function graphed? graph parabola -> function quadratic

This graphed shape is called a **parabola** and the function modeled is known as a **quadratic function.** What is the domain of a quadratic function?

The function in standard form is:

$f(x) = ax^2$	$+\, bx$	$+$	c
quadratic term	linear term		constant term

A quadratic function will have this form. Yes, "b" can equal 0 and "c" can equal 0. <u>If "a" equals 0 then it is no longer a quadratic, but linear.</u> parent $y = x^2$

Axis of symmetry – a line that divides a parabola into two parts that are mirror images of each other. The axis of symmetry will be a vertical line with an equation in the form of **x=real number** and will be equal to the **x** value.

Vertex – is where the minimum or maximum value of the function and will occur at the value of the **y point**. This is the point where the direction of the parabola changes from decreasing to increasing or increasing to decreasing. vertex (x,y) min or max y = #

Minimum or Maximum – the value of y at the vertex

Corresponding point – points on a parabola that are the reflection of other points on the parabola.

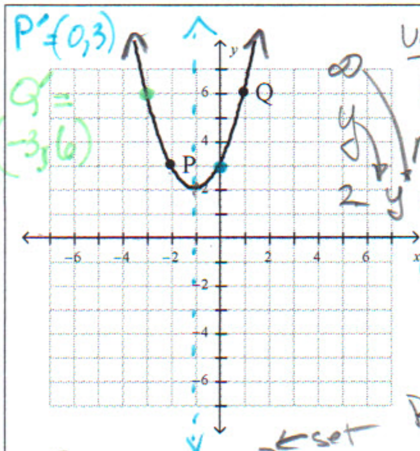
e.g. on the above graph $P\,(-2,-1)$ *corresponds to* P' ((0, −1), plot P', What is Q'?

Identify the vertex, minimum or maximum, axis of symmetry and the domain and range for the graphs.

Identify the corresponding points **for P and Q**

P' (0, 3)

Q' = (-3, 6)



vertex (-1, 2)

min y = 2

2 ≤ y < ∞

a.o.s. x = -1

R: {2 ≤ y < ∞} ← set

[2, ∞)

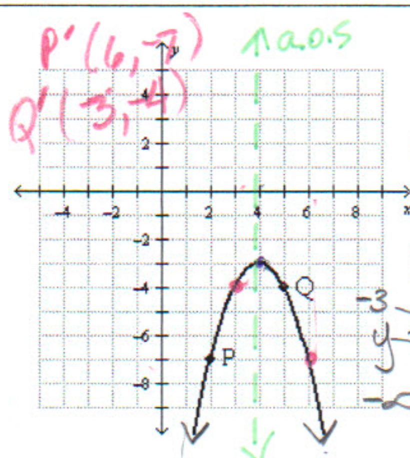
D: {-∞ < x < ∞}, all Reals

(-∞, ∞)

P' (6, -7)

Q' (3, -4)

a.o.s



Vertex (4, -3)

a.o.s x = 4

maximum y = -3

-∞ < y ≤ -3

R: {-∞ < y ≤ -3}

Int: (-∞, -3]

D: {set of All Real #}

Set: {-∞ < x < ∞}

Interval (-∞, ∞)

If a calculator is allowed, you may find the minimum/maximum point of the parabola:

1. Using the **Y=** button enter the equation.
2. Hit **GRAPH** ***Note: the stat plots must be off for the graphing function to work.***
3. Adjust the window of the view screen under **ZOOM** or **WINDOW** in order to view the vertex.
4. Hit **2nd TRACE 3** minimum or **4** maximum. The view screen will ask for the left bound, arrow over so that the blinking star (asterisk) is on the left side of the vertex. **ENTER** You will be asked for the right bound, and again arrow over so that the asterisk is now on the right side of the vertex. **ENTER ENTER** and the view screen will identify the x and y coordinates for the vertex.

Example: Use the calculator to write a quadratic equation with the following points:

x	2	3	4
y	3	13	29

$a = 3,\ b = -5,\ c = 1$

$\Rightarrow y = 3x^2 - 5x + 1$

$(1, -2), (2, -2), (3, -4)$

$a = -1 \quad b = 3 \quad c = -4$

$y = -x^2 + 3x - 4$

Here we use different methods to solve:

$\underbrace{y = ax^2 + bx + c}_{x-y \text{ values}}$

Method 1:

1. A system of 3 equations and 3 unknowns may be solved with elimination or substitution.

Method 2:

1. Write as a matrix equation and solve.

Method 3:

1. Use the stat plot function on the calculator to plot the original given points
2. A quadratic regression (calculator) will yield the equation of best fit.

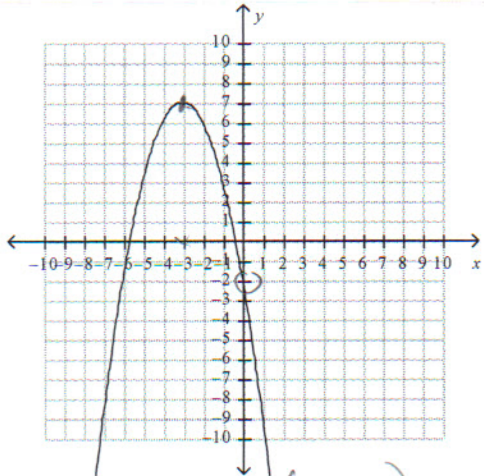
Method 1 and 2:

Given 3 ordered pairs, a quadratic equation may be found:
1. Substitute the values of x and y into the quadratic equation: $y = ax^2 + bx + c$ 2. With the 3 resultant equations you have a **system of 3 linear equations** and may be solved by methods learned in Chapter 3 and 4 or in the "Final Word on Chapter 4" lesson. 3. Using the augmented matrix form, key in the coefficients and constant into a 3x4 matrix on the calculator and find the reduced row-echelon form of the matrix, thus finding the solutions to the variables which are in fact the coefficients of the quadratic equation!

Method 3:

LINEAR/QUADRATIC REGRESSION	
Using STAT PLOT and finding a best fit line or curve:	
1. Given a set of data: 2. Enter the Data into the calculator:	**[STAT] [ENTER]** type in independent variable **(x)** data into **L1**, dependent data **(y)** data into **L2,** followed by **2^{nd} [MODE]** (quit)
3. Turn on STAT PLOT1	**2^{nd} [Y=] [Enter] [Enter] 2^{nd} [MODE]** **OR** **[Y=] ↑ Plot1 [Enter]**
4. Plot the data points	**[ZOOM] – 9**
5. Find the best fit and send the equation over to the y-plot select **4** for linear or **5** for quadratic regression.	**[STAT] [CALC] 5 [VARS] ► Y-VARS [ENTER] [ENTER] [ENTER] [GRAPH]**
6. When **Y=** is opened you will see the equation has been placed for graphing, and a line will be drawn of best fit	***** IF YOU ONLY NEED AN EQUATION:** **STAT ► CALC 4 ENTER** FOR A LINEAR **STAT ► CALC 5 ENTER** FOR A QUADRATIC

Even with the limited knowledge we have from just completing one section, we can still come up with some information that describes a graph of a quadratic and choose an equation that represents the function modeled.



What is the vertex? (-3,7)

What is the y-intercept? y=-2

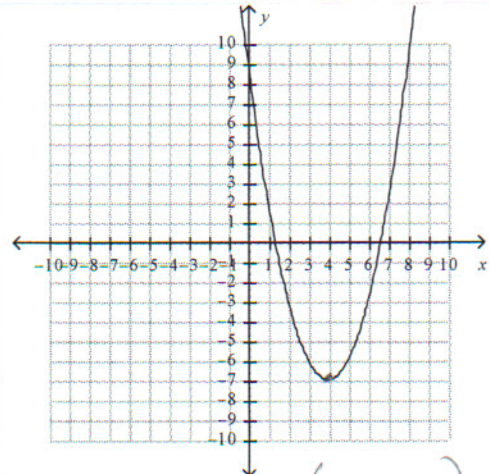
Leading coefficient, + / - ? −

a. $f(x) = \cancel{-x^2} + 6x + 2$

b. $f(x) = -x^2 - 6x - 2$?

c. $f(x) \cancel{= -x^2} - 6x + 2$

d. $f(x) = -x^2 + 6x - 2$?



What is the vertex? (4,-7)

What is the y-intercept? (+9)

Leading coefficient, + / - ?

a. $f(x) = x^2 - 8x - 9$

b. $f(x) = x^2 + 8x + 9$

c. $f(x) = x^2 + 8x - 9$

d. $f(x) = x^2 - 8x + 9$

b) $\overset{y}{7} = -(-3^2) - 6(-3) - 2$

$7 \overset{?}{=} -9 + 18 - 2$ ✓

b) $-7 \overset{?}{=} 4^2 + 8(4) + 9$

$= 16 + 32 + 9$

not $= -7$!

d) $-7 = 4^2 - 8(4) + 9$

$= 16 - 32 + 9$

$= -7$ ✓

Algebra 2

Lesson 5-2: Properties of Parabolas

Mrs. Snow, Instructor

PARABOLA STANDARD FORM:

$\boldsymbol{y = ax^2 + bx + c}$

1. When b=0, the function is: $y = ax^2 + c$. When graphed, the parabola will be symmetric around the y-axis. Therefore, the **axis of symmetry is: $\boldsymbol{x = 0}$**, and the **vertex of the graph is the y-intercept, (0,c).**
2. If a>0 the parabola will open upward. a<0, open downward.
3. The larger **a,** the narrower the parabola. The smaller **a,** the wider the parabola.
4. Setting $x = 0 \rightarrow y = c \quad \therefore \quad c\ is\ the\ y - intercept!$

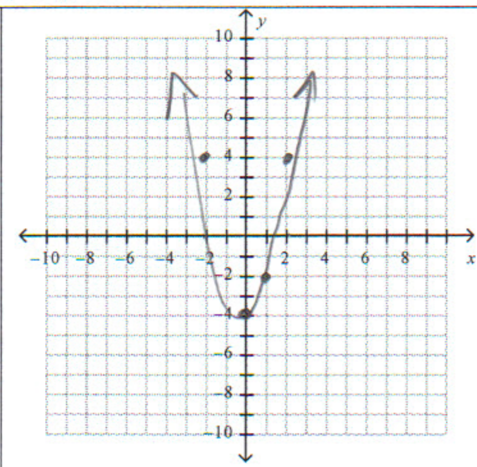
To graph a quadratic equation in the form $\boldsymbol{y = ax^2 + c}$:

1. The vertex is at (0,c). Note that this is also the y-intercept.
2. The sign of **"a"** tells us it opens up **(+)** or down **(-).**
3. Pick at least 3 points on one side of the vertex, solve for y and then find the **corresponding** points using symmetry to graph the other side.

Graph the function $y = 2x^2 - 4$

$x = 0$
$y = -4$

	x	y	
$2(-2^2)-4$	-2	4	
	0	-4	(v)
$2(1^2)-4$	1	-2	
$2(2^2)-4$	2	4	



Well, what if the **equation is in standard form: $\boldsymbol{y = ax^2 + bx + c}$?** ← y - inpt

1. The sign of the coefficient of **a** still tells us whether the parabola opens up **(+)** or down **(-).**
2. Axis of symmetry is now found from the coefficients of the equation, hence the axis is the line: $\boldsymbol{x = \frac{-b}{2a}}$
3. The vertex of the parabola is at the point: $\boldsymbol{x = \frac{-b}{2a},\ y = f\left(\frac{-b}{2a}\right)}$;

   $basically\ when\ \boldsymbol{x} = \frac{-b}{2a}, what\ is\ y?$
4. Now, the parabola will be translated along the x-axis; however , the **y-intercept** is at **(0,c)**.

Example: Graph the function: $y = 3x^2 + 6x - 4$

1. $a = 3 \quad b = 6 \quad c = -4$
2. a *is* $+$ *or* $-$ *so opens* up
3. $y = intercept = c =$ -4

   plot point
4. Axis of symmetry: $x = \frac{-b}{2a} = \frac{-6}{2(3)} = \frac{-6}{6}$

   $-\mathbf{1} = \boldsymbol{x}$ graph axis of symmetry.
5. Vertex: $x = -1$: $y = f\left(\frac{-6}{2(3)}\right) =$

   $f(-1) = -7$

   When $x = -1$, solve for y

   $y = 3(-1)^2 + 6(-1) - 4$

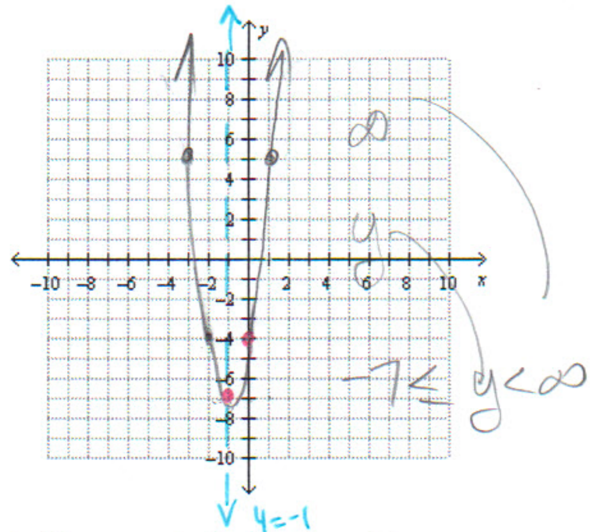
   $= 3 - 6 - 4 = -7$

   $\therefore$ $(-\mathbf{1}, -\mathbf{7}), \boldsymbol{vertex}.$
6. Select x-values adjacent to the axis of symmetry and find the corresponding y=value

$y = 3x^2 + 6x - 4$

x	-4	-3	-2	-1	0	1	2
y		5	-4	-7	-4	5	

7. Complete table with corresponding points reflected across the axis of symmetry.



8. The vertex is the location of the minimum/maximum.

   What is this value? $y = -7$

9. Domain: All Real

   Range: $\{7 \le y < \infty\}$

   $[-7, \infty)$

$3(1)^2 + 6(1) - 4$

$3 + 6 - 4$

$3(-3^2) + 6(-3) - 4$

$27 - 18 - 4$

$9 - 4$

Calculator Minimums and Maximum:

1. Hit **Y=** type in the quadratic equation. **Remember:** must be in the "y=" form.
2. **GRAPH** if the parabola is off the view screen: **WINDOW** adjust the minimum and maximum values for x and y. **GRAPH** and view the parabola.
3. **2nd TRACE** choose **3-minimum** if the parabola is opening up or choose **4-maximum** if the parabola is opening down. Question: **left bound?** Arrow over so that asterisk is flashing on the left side of the min or max **ENTER right bound?** Arrow over so that the asterisk is flashing on the right side of the min or max. **ENTER Guess? ENTER** the x and y values will be given at the bottom of the view screen.

Given the equation: $y = -x^2 + 2x + 3$

1. $a =$ -1 $b =$ 2 $c =$ 3
2. a is + or − so opens ______
3. $y = intercept = c =$ 3

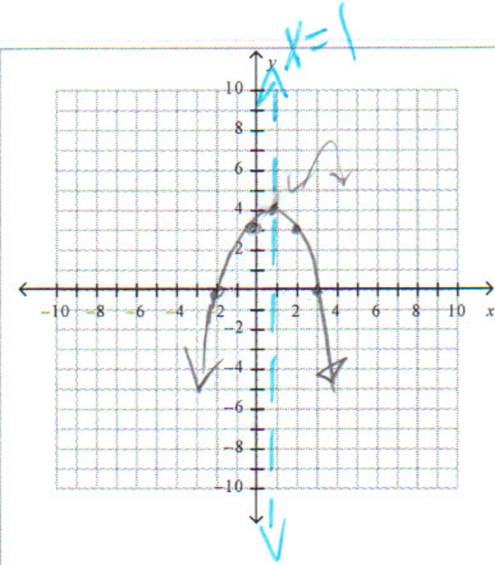
   plot point
4. Calculate and graph Axis of symmetry: $x = \frac{-b}{2a}$ $= \frac{-2}{2(-1)}$ $\frac{-2}{-2}$ $= 1$
5. Calculate vertex

   $y = -(1^2)+2(1)+3$

   $-1+2+3 = 4$
6. Make a table of values including y-intercept and vertex Choose 3 points on one side of the vertex. (1, 4)

x	y
0	3
1	4 ✓
2	3
3	0

7. Compete the table of values and graph.



8. State the minimum/maximum value.

9. Domain: {ℝ}

Range: $-\infty < y \leq 4$

$(-\infty, 4]$

$-(3^2)+2(3)+3$

$-9+6+3$

Application:

The number of bacteria in a refrigerated food is given by $n(t) = 20t^2 - 20t + 120$, for $-2 \le t \le 14$ and where t is the temperature of the food in Celsius. At what temperature will the number of bacteria be a minimum?

t temperature

parabola opens up so minimum

vertex (temp, bacteria)

temperature independent ⇒

$$t = \frac{-b}{2a} = \frac{-(-20)}{2(20)} = \frac{20}{40} = \frac{1}{2}$$

⇒ so bacteria at minimum when $\underline{t = \frac{1}{2}°C}$

Nike Shoes estimates that its monthly profit P in hundreds of dollars can be modeled by the formula $P = -2x^2 + 4x + 6$, where x is the number of shoes produced per month in thousands. How many shoes should be produced per month to earn the maximum profit? What is the maximum monthly profit?

(x, y) = (shoes, profit)

Leading coefficient = −2

Parabola opens down

Vertex at Maximum

$$x = \frac{-b}{2a} = \frac{-4}{2(-2)} = \frac{-4}{-4} = 1$$

∴ $\underline{x = 1000 \text{ shoes}}$

$P = -2(1) + 4(1) + 6$

$= -2 + 4 + 6$

$= 8$ OR $\underline{\$800 \text{ profit}}$

A company's weekly revenue in dollars is given by $R(x) = 2000x - 2x^2$, where x is the number of items produced during a week. What amount of items will produce the maximum revenue?

leading coefficient

$a = -2$ parabola opens down

max at vertex

$$x = \frac{-b}{2a} = \frac{-2000}{2(-2)} = \frac{-2000}{-4} = \boxed{500 \text{ number of items}}$$

note: max revenue at x = 500

$R(x) = 2000(500) - 2(500^2)$

$= \underline{\$500{,}000 \text{ revenue}}$