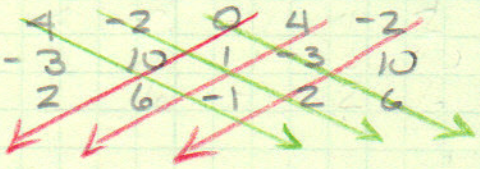
1. Write matrix as a determinant then repeat col 1 & col 2
2. Sum the down to the right products
3. Sum the "down to the left products
4. Subtract

$$A = \begin{bmatrix} 4 & -2 & 0 \\ -3 & 10 & 1 \\ 2 & 6 & -1 \end{bmatrix} \quad \det A = \begin{vmatrix} 4 & -2 & 0 \\ -3 & 10 & 1 \\ 2 & 6 & -1 \end{vmatrix}$$ solve

repeat col 1 & col 2



$(4 \cdot 10 \cdot -1) + (-2 \cdot 1 \cdot 2) + (0 \cdot -3 \cdot 6) = -44$

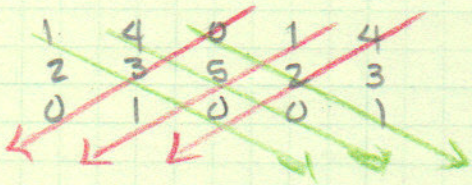
minus

$(2 \cdot 10 \cdot 0) + (6 \cdot 1 \cdot 4) + (-1 \cdot -3 \cdot -2) = -18$

-62

$\det A = -62$

$$A = \begin{bmatrix} 1 & 4 & 0 \\ 2 & 3 & 5 \\ 0 & 1 & 0 \end{bmatrix} \quad \det A = \begin{vmatrix} 1 & 4 & 0 \\ 2 & 3 & 5 \\ 0 & 1 & 0 \end{vmatrix}$$



$(1 \cdot 3 \cdot 0) + (4 \cdot 5 \cdot 0) + (0 \cdot 2 \cdot 1) = 0$

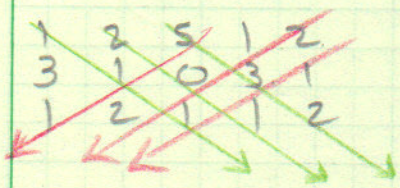
minus

$(0 \cdot 3 \cdot 0) + (1 \cdot 5 \cdot 1) + (0 \cdot 2 \cdot 4) = 5$

-5

$\det A = -5$

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad \det A = \begin{vmatrix} 1 & 2 & 5 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix}$$



$(1 \cdot 1 \cdot 1) + (2 \cdot 0 \cdot 1) + (5 \cdot 3 \cdot 2) = 31$

Minus

$(1 \cdot 1 \cdot 5) + (2 \cdot 0 \cdot 1) + (1 \cdot 3 \cdot 2) = 11$

20

$\det A = 20$

Solve for matrix X (calculator)

$$A^{-1}\begin{bmatrix} 0 & 0 & 2 \\ 1 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} X = A^{-1}\begin{bmatrix} 6 \\ -11 \\ 8 \end{bmatrix}$$

Remember! No commutative multiplication property in matrices

Order Counts

calculator:

$$A^{-1} = \begin{bmatrix} .1 & .4 & .6 \\ .3 & .2 & -.2 \\ .5 & 0 & 0 \end{bmatrix} \quad \text{so: } X = \begin{bmatrix} .1 & .4 & .6 \\ .3 & .2 & -.2 \\ .5 & 0 & 0 \end{bmatrix}\begin{bmatrix} 6 \\ -11 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

See 4.6 lesson for directions on calculator.

4.7 Matrix Systems

A system of equations may be written in matrix form: Separate each part into a matrix

coefficient variable constant

$$\begin{cases} 1x + 2y = 5 \\ 3x + 5y = 14 \end{cases} \quad \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 14 \end{bmatrix}$$

so solve the system with matrices:

$$\begin{cases} 2x + 3y = 11 \\ x + 2y = 6 \end{cases} \quad A^{-1} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} 11 \\ 6 \end{bmatrix}$$

$\det A = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1$

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 11 \\ 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$I \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 - 18 \\ -11 + 12 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$\therefore \underline{(4, 1)}$ check $2(4) + 3(1) = 11$ yes

$4 + 2(1) = 6$ yes

System 3 equations? Yes use a calculator to find A^{-1}

$$\begin{aligned} 2x + y + 3z &= 1 \\ 5x + y - 2z &= 8 \\ x - y - 9z &= 5 \end{aligned}$$

order counts!!

$$A^{-1} \begin{bmatrix} 2 & 1 & 3 \\ 5 & 1 & -2 \\ 1 & -1 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} 1 \\ 8 \\ 5 \end{bmatrix}$$

approx A^{-1} on calculator – you will have to toggle right to get col 2 & 3

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3.\overline{6} & 2 & -1.\overline{6} \\ 14.\overline{3} & -7 & 6.\overline{3} \\ -2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 8 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \\ 1 \end{bmatrix}$$

$\underline{(4, -10, 1)}$

Check an eqn

$2(4) - 10 + 3(1) = 1$?

$8 - 10 + 3 = 1$ ✓