

## Algebra 2

### Lesson 4-2: Adding and Subtracting Matrices

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Plain and simple, matrices are rectangular arrangements of numbers. These numbers may be data or they may be numbers from an equation. Today, matrices are used in many areas of work and research. Some of the many areas include electrical circuits, battery power outputs, and 2-dimensional and 3-dimensional screen images that create realistic movements. In geology matrices are used in seismic surveys. Real world data like banking, profit and loss, movie ticket sales, and on a big level, gross domestic profit! The base element for robot movement is matrices.

#### Vocabulary

**matrix** - rectangular arrangement of numbers in rows and columns

**element** - each individual number in a matrix is an element

**dimensions** - describes the number of rows and columns that are the given matrix

**ROW X COLUMN - R X C** - This tells us how many rows and how many columns. An element has an **address** in the form of **RC**.

**Rules for Matrix Addition/Subtraction:**

1. To add two matrices, they must have the same dimensions.
2. Add corresponding elements together.
3. A number outside the brackets is scalar multiplication treat it as distribution to all elements

$$2 \times 3 \Rightarrow \begin{matrix} 2 \text{ rows} \\ 3 \text{ columns} \end{matrix} \begin{bmatrix} 1,1 & 1,2 & 1,3 \\ 2,1 & 2,2 & 2,3 \end{bmatrix}$$

#### Example

Find the sum or difference

$$\begin{bmatrix} 2 & -4 & 3 \\ 0 & 1 & 12 \end{bmatrix} + \begin{bmatrix} 0 & 7 & -2 \\ 13 & -1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 13 & 0 & 17 \end{bmatrix}$$

$$2(x+7) = 2x+14 \quad 5+x - (2+2x) = 3-x$$

$$2 \begin{bmatrix} 6 & 5 \\ 10 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 3 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 10 \\ 20 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 3 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 7 \\ 20 & 1 \end{bmatrix}$$

#### Properties of Matrix Addition:

If A, B, and C are m x n matrices (m rows and n columns) then:

- |   |                                  |
|---|----------------------------------|
| 1. $A + B$ is an m x n matrix                                       | Closure Property                 |
| 2. $A + B = B + A$  | Commutative Property of Addition |
| 3. $(A + B)C = A + (B + C)$   | Associative Property of Addition |
| 4. There is a unique matrix O such that $O + A = A + O = A$         | Additive Property of Addition    |
| 5. For each A there exists a unique opposite, $-A$ , $A + (-A) = O$ | Additive Inverse Property        |

## Matrix equations

Matrix equations are solved much like an equation in one variable, only the variable is an unknown matrix and the numbers are matrices!!

Capital letter  $\Rightarrow$  Matrix Name

Solve

<p>Algebra</p> $x - 6 = 13$ $+6 \quad +6$ $x = 19$	<p><math>3 \times 2</math></p> $+ \begin{bmatrix} 2 & 1 \\ 0 & -4 \\ 5 & 3 \end{bmatrix} X - \begin{bmatrix} 2 & 1 \\ 0 & -4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 13 \\ -12 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & -4 \\ 5 & 3 \end{bmatrix}$ $X = \begin{bmatrix} 7 & 8 \\ 2 & 9 \\ -7 & 3 \end{bmatrix}$
$\begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -10 & 2 \end{bmatrix} - X$ $- \begin{bmatrix} 4 & 5 \\ -10 & 2 \end{bmatrix} = - \begin{bmatrix} 4 & 5 \\ -10 & 2 \end{bmatrix}$ $- \begin{bmatrix} +2 & +4 \\ -10 & +5 \end{bmatrix} = \cancel{(-X)} X$ $\begin{bmatrix} 2 & 4 \\ -10 & 5 \end{bmatrix} = X$	<p><math>3 \times 1</math></p> $\begin{bmatrix} 6 \\ -8 \\ -6 \end{bmatrix} 2X + \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$ $- \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ $\left(\frac{1}{2}\right) 2X = \begin{bmatrix} 6 \\ -8 \\ -6 \end{bmatrix} \left(\frac{1}{2}\right)$ $X = \begin{bmatrix} 3 \\ -4 \\ -3 \end{bmatrix}$

Finding the value of a variable

$\begin{bmatrix} a & 2b \\ c-2 & d+3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ 10 & 10 \end{bmatrix}$ $\underline{a = 5} \quad 2b = -7 \quad c-2 = 10 \quad d+3 = 10$ $b = -\frac{7}{2} \quad \underline{c = 12} \quad \underline{d = 7}$	$\begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -13 \\ -10 & -10 \end{bmatrix} = \begin{bmatrix} 2x+1 & -4x \\ 5z & 2.5z-x \end{bmatrix}$ $\begin{bmatrix} 7 & -12 \\ -10 & -8 \end{bmatrix} = \begin{bmatrix} 2x+1 & -4x \\ 5z & 2.5z-x \end{bmatrix}$ $\left(-\frac{1}{4}\right) 12 = -4x \left(-\frac{1}{4}\right) \quad \left(\frac{1}{5}\right) 5z = -10 \left(\frac{1}{5}\right)$ $3 = x \quad \underline{z = -2}$
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Multiply the matrices

$$\begin{bmatrix} 3 & 2 & 0 \\ -1 & 4 & -2 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 0 \\ -2 & 3 & 2 \\ 1 & 0 & -3 \end{bmatrix} \quad \text{check: } 2 \times \underbrace{3 \text{ times } 3}_{\text{match}} \times 3$$

product dimensions =  $2 \times 3$

set up a template to fill in values:

$$= \begin{bmatrix} \begin{array}{l} \text{red} \rightarrow \text{blue} \downarrow \\ (3)(1) + (2)(2) + (0)(1) \\ (1)(1) \end{array} & \begin{array}{l} \text{red} \rightarrow \text{green} \downarrow \\ (3)(4) + (2)(3) + (0)(0) \\ (1)(2) \end{array} & \begin{array}{l} \text{red} \rightarrow \text{red} \downarrow \\ (3)(0) + (2)(2) + (0)(-3) \\ (1)(3) \end{array} \\ \begin{array}{l} \text{pink} \rightarrow \text{blue} \downarrow \\ (-1)(1) + (4)(2) + (-2)(1) \\ (2)(1) \\ -1 - 8 - 2 \end{array} & \begin{array}{l} \text{pink} \rightarrow \text{green} \downarrow \\ (-1)(4) + (4)(3) + (-2)(0) \\ (2)(2) \\ -4 + 12 + 0 \end{array} & \begin{array}{l} \text{pink} \rightarrow \text{red} \downarrow \\ (-1)(0) + (4)(2) + (-2)(-3) \\ 2(3) \\ 0 + 8 + 6 \end{array} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 18 & 4 \\ -11 & 8 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ -4 & 3 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 7 & 0 \\ -2 & 3 & 1 \end{bmatrix}$$

$$(3 \times \underbrace{2}_{\text{match}})(2 \times 3)$$

3x3 product

$$\begin{array}{l} \text{red} \rightarrow \text{blue} \downarrow \\ (1)(5) + 2(-2) \\ 1, 1 \\ \text{pink} \rightarrow \text{blue} \downarrow \\ (-4)(5) + 3(-2) \\ 2, 1 \\ -20 - 6 \\ (0)(5) + 1(-2) \downarrow \\ 3, 1 \rightarrow \\ 0 - 2 \end{array}$$

$$\begin{array}{l} \text{red} \rightarrow \text{red} \downarrow \\ (1)(7) + 2(3) \\ 1, 2 \\ \text{pink} \rightarrow \text{green} \downarrow \\ (-4)(7) + (3)(3) \\ 2, 2 \\ (0)(7) + (1)(3) \rightarrow \text{red} \downarrow \\ 3, 2 \rightarrow \end{array}$$

$$\begin{array}{l} \text{red} \rightarrow \text{red} \downarrow \\ (1)(0) + (2)(1) \\ 1, 3 \\ \text{pink} \rightarrow \text{red} \downarrow \\ (-4)(0) + (3)(1) \rightarrow \text{pink} \downarrow \\ 2, 3 \\ (0)(0) + (1)(1) \rightarrow \text{pink} \downarrow \\ 3, 3 \end{array}$$

$$\begin{bmatrix} 1 & 13 & 2 \\ -26 & -19 & 3 \\ -2 & 3 & 1 \end{bmatrix} \quad \underline{\underline{3 \times 3}}$$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 \\ 1 & 6 \end{bmatrix}$$

$$(\cancel{2 \times 2})(\cancel{2 \times 2})$$

2x2 prod.

$$\begin{array}{r} \xrightarrow{\quad} \downarrow \\ (1)(4) + 0(1) \\ \hline 1 \quad 1 \\ (2)(4) + (1)(1) \\ \hline 2 \quad 1 \end{array}$$

$$\begin{array}{r} \xrightarrow{\quad} \downarrow \\ (1)(-2) + 0(6) \\ \hline 1 \quad 2 \\ (2)(-2) + (1)(6) \\ \hline 2 \quad 2 \end{array}$$

$$= \begin{bmatrix} 4 & -2 \\ 9 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 0 \\ 1 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix}$$

$$(\cancel{2 \times 3})(\cancel{2 \times 3})$$

No Match

Not possible  
Dimension  
error