

3.6 - Systems with 3 Variables

$$x - y - z$$

Elimination

#1 Eliminate a variable - combine 2 eqs.

#2 Eliminate same variable w/ 3rd eqn and one other eqn.

$$\begin{cases} \textcircled{1} & x + y + z = 6 \\ \textcircled{2} & 2x - y + 3z = 9 \\ \textcircled{3} & -x + 2y + 2z = 9 \end{cases}$$

$$\begin{array}{r} \text{Eq 1 \& 2: } x + y + z = 6 \\ \quad 2x - y + 3z = 9 \\ \hline 3x + 4z = 15 \end{array}$$

$$\begin{array}{r} \text{Eq 2 \& 3: } 2(2x - y + 3z) = 9(2) \\ \quad -x + 2y + 2z = 9 \\ \hline 4x - 2y + 6z = 18 \\ \quad \quad \quad 3x + 8z = 27 \end{array}$$

↔
2 equations
2 variables

$$\begin{array}{r} -1(3x + 4z) = 15(-1) \rightarrow -3x - 4z = -15 \\ \quad 3x + 8z = 27 \\ \hline 4z = +12 \end{array}$$

$$\begin{array}{r} 3x + 4(\frac{3}{2}) = 15 \\ 3x + 12 = 15 \\ 3x = 3 \\ \underline{x = 1} \end{array}$$

$$\begin{array}{r} x + y + z = 6 \\ 1 + y + 3 = 6 \\ 4 + y = 6 \\ \underline{y = 2} \end{array}$$

$$\begin{array}{r} (x, y, z) \\ \underline{(1, 2, 3)} \end{array}$$

Solution

Always check work—

Substitution

- + Solve equation for a variable
- + Substitute above term into remaining equations.
- + Write the 2 equations as a system as outlined in 3.2 (sys of 2 eqs) & solve.

$$x + y + z = -8$$

$$x - y - z = 6 \Rightarrow x = y + z + 6$$

$$2x - 3y + 2z = -1$$

$$\begin{array}{r} x \\ y+z+6 \end{array} + y + z = -8 \Rightarrow 2y + 2z + 6 = -8$$
$$2y + 2z = -14 \leftarrow$$

$$2(y + z + 6) - 3y + 2z = -1$$

$$\begin{array}{r} 2y + 2z + 12 - 3y + 2z = -1 \\ -y + 4z = -13 \end{array}$$

$$\Rightarrow (2)y + 4z = -13 \quad (2)$$

$$\begin{array}{r} 2y + 2z = -14 \\ -2y + 8z = -26 \\ \hline 10z = -40 \end{array}$$

$$\underline{\underline{z = -4}}$$

$$x + y + z = -8$$

$$x + -3 - 4 = -8$$

$$x + (-7) = -8$$

$$x = -1$$

$$\underline{\underline{(-1, -3, -4)}}$$

$$2y + 2(-4) = -14$$

$$2y - 8 = -14$$

$$2y = -6$$

$$\underline{\underline{y = -3}}$$