Algebra 2
Lesson 3-4: Linear Programming
Mrs. Snow, Instructor

When the United States entered World War II, it quickly became apparent to the U.S. leaders in order to win the war, massive amounts of resources would be required, and building/upgrading of many factories would be a priority. Food, planes, medicine, tanks, and other weaponry had to be transported to many places worldwide. The problem was how could the United States transport massive amounts of supplies and weapons to key points worldwide in a short time?

Dr. George Dantzig of Stanford University had the answer, but his methodology worked only for a few equations and was limited to a few variables. Though his contributions to the war effort were significant, his **Simplex** method was not fully explored until the 1950's when computer technology could operate hundreds of variables and equations. **Simplex** is now known as **Linear Programming**.

Today linear programming allows industries and companies to determine the best solution to an economic decision. In other words, how they can maximize profits and minimize costs. Linear programming is most commonly seen in operations research because it provides a "best" solution, while considering all the constraints or limitations of the situation

Constraints – a system of linear equations that place restrictions on the situation. E.g. how much of a certain item can be made in how much time.

Feasibility Region – the resulting "walled off" area formed by the constraints.

Objective Function (optimization equation) – an equation that models a situation that you want to determine either the maximum or minimum quantity.

Maximum or Minimum values – these are the optimal values and occur at the vertices of your feasibility region.

Rule: ***If there is a maximum or a minimum value of the linear objective function, it occurs at one or more vertices of the feasible region.***

Process to find Maximum or Minimum values:

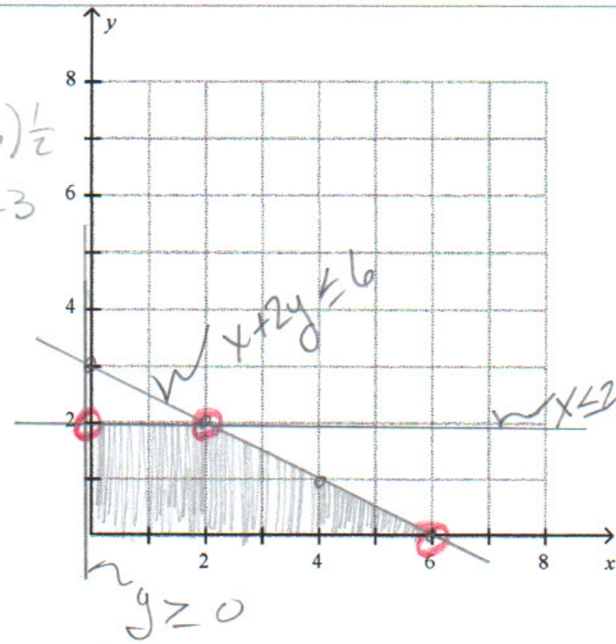
1. If required, determine the inequalities that describe the constrains
2. Graph the inequalities, shading the feasibility region
3. ID and list the vertices of the feasibility area: *the maximum/minimum value will be one of these points.*
4. Plug the vertex points into the objective function and solve.
5. Select the (x,y) point that gives the required maximum/minimum output/

Graph the system of constraints.
Find the values of x and y that maximize or minimize the objective function.

$$\begin{cases} x + 2y \le 6 \\ x \le 2 \\ y \ge 0 \end{cases}$$ **graph**

$\frac{1}{2}(2y) \le (-x+6)\frac{1}{2}$

$y \le -\frac{1}{2}x + 3$



Objective function:

$minimum\ for: C = 3x + 4y$

List (x,y) vertices:

(0,2), (2,2) (6,0)

Calculate minimum

$C = 3(0) + 4(2) = 8$

$C = 3(2) + 4(2) = 6 + 8 = 14$

$C = 3(6) + 4(0) = 18$

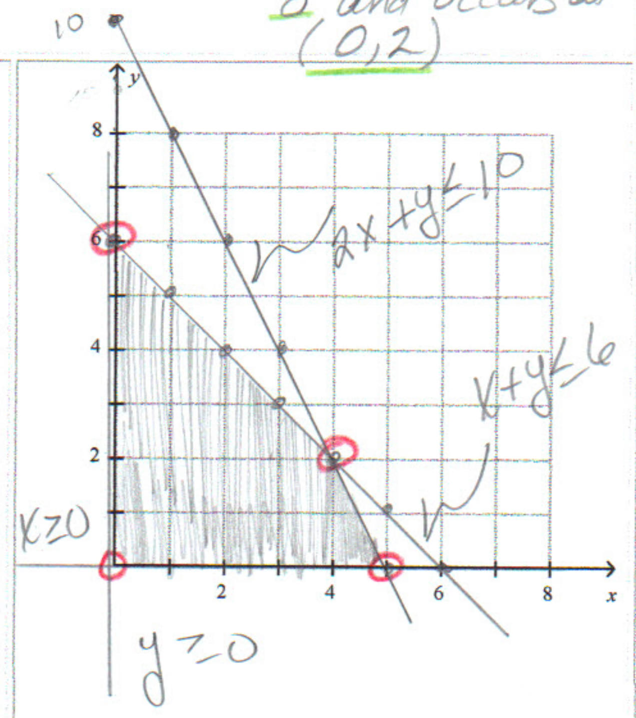
Ans: minimum is 8 and occurs at (0,2)

Graph:

$$\begin{cases} x + y \le 6 \\ 2x + y \le 10 \\ x \ge 0, \quad y \ge 0 \end{cases}$$

$y \le -x + 6$

$y \le -2x + 10$



Objective function:

$maximum\ for:\ P = 4x + y$

(0,6) (4,2) (5,0)

$P = 4(0) + 6 = 6$

$P = 4(4) + 2 = 18$

$P = 4(5) + 0 = 20$

Maximum is 20
Occurs at (5,0)

The Swiss Chocolate Company makes two types of chocolates: dark chocolate and white chocolate. They can produce a total of 240 cases of chocolates. Based on previous sales, the company must make at least 60 cases of dark chocolate. They must produce 40 cases of white chocolate but no more than 160 cases. They can sell each case of dark chocolate for $12.50 and each case of white chocolate for $17.50. <u>How many cases of each type of chocolate should they produce to maximize revenue?</u>

Constraints are linear inequalities:

$let\ x = dark\ chocolate\ and\ y = white\ chocolate$

Facts:

- ***They can produce no more than 240 total cases***
- ***Must produce at least 60 cases of dark chocolate***
- ***Must produce at least 40 cases of white chocolate***
- ***Cannot produce more than 160 cases of white chocolate***
- ***Objective function calculates the total revenue:***

$x + y \leq 240$ $\quad y \leq -x + 240$

$x \geq 60$

$y \geq 40$

$y \leq 160$

Objective function: $\quad$ Units Revenue ⇒ $

$P = 12.5x + 17.5y$

(60, 160) (200, 40)

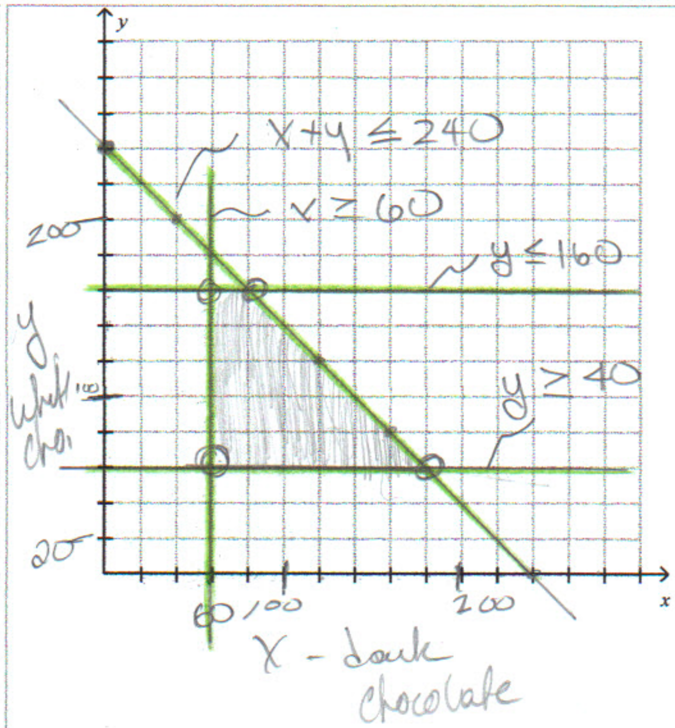
(80, 160) (60, 40)

P = 12.5(60 + 17.5(160)
= 750 + 2800 = $3550

P = 12.5(80) + 17.5(160)
= 1000 + 2800 = $3800

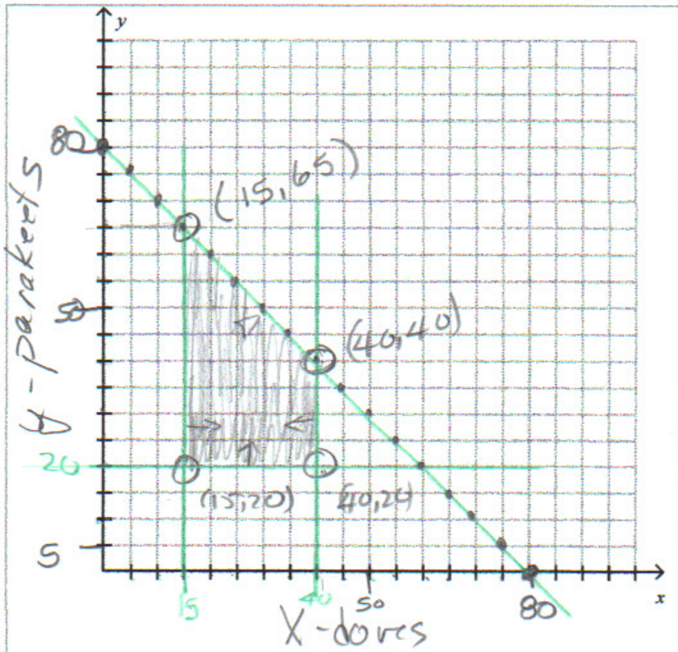
P = 12.5(200) + 17.5(40)
= 2500 + 700 = $3200

P = 12.5(60) + (7.5)(40)
= 750 + 700 = $1450



Maximum Revenue $3800
When we produce
80 dark chocolate
160 white chocolate

A local bird breeder can produce 80 chicks per year. He specializes in breeding doves and parakeets. the demand for his birds requires that he have between 15 and 40 doves and at least 20 parakeets. He sells the parakeets for \$7.50 each and sells the doves for \$8.25 each. <u>How many of each type of bird should he produce in order to maximize revenue?</u> Money



let x= doves - y= parakeets

Constraints:

1. $x + y \leq 80 \rightarrow y \leq -x + 80$
2. $x \geq 15 \quad x \leq 40$
3. $y \geq 20$

4.objective function: $P = 8.25x + 7.50y$

(15, 20) (15, 65) (40, 40)

(40, 20) ← put into obj function

$P = 8.25(15) + 7.50(20) = \$277.50$

$P = 8.25(15) + 7.5(65) = \$611.25$

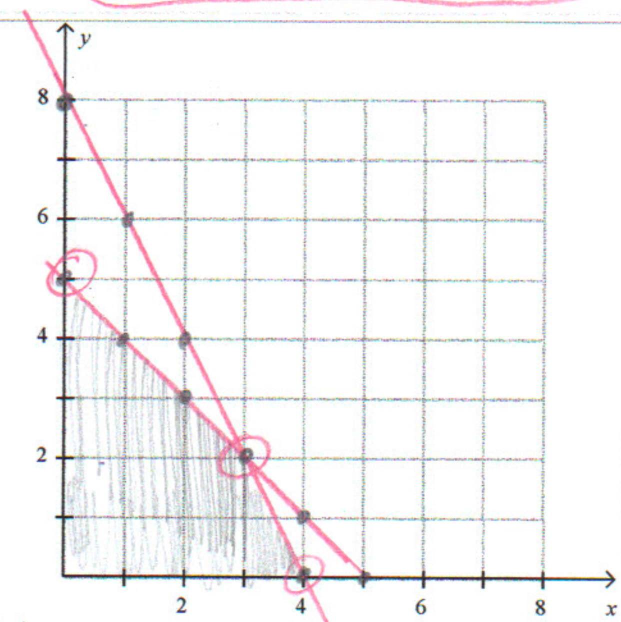
$P = 8.25(40) + 7.5(40) = \$630$

$P = 8.25(40) + 7.5(20) = \$480$

Max profit \$630.00

40 doves & 40 parakeets

Baking a tray of corn muffins takes 4 cups of milk and 3 cups of flour. A tray of bran muffins takes 2 cups of milk and 3 cups of flour. A baker has 16 cups of milk and 15 cups of four. He makes a \$30 profit per tray of corn muffins and \$20 profit for every tray of bran muffins. <u>How many trays of each type of muffin should the baker make in order to maximize his profit?</u>



let x= corn muff. - y= bran muff

Constraints:

	Milk	flour
corn muffins	4	3
bran muffins	2	3
have	16	15

$4x + 2y \leq 16$

$\frac{1}{2}(2y) \leq (-4x + 16)\frac{1}{2}$

$y \leq -2x + 8$

$3x + 3y \leq 15$

$\frac{1}{3}(3y) \leq (-3x + 15)\frac{1}{3}$

$y \leq -x + 5$

& $x \geq 0 \quad y \geq 0$

objective function: $30x + 20y = P$

(0,5), (3,2), (4,0)

$P = 30(0) + 20(5) = \$100$

$P = 30(3) + 20(2) = \$130$

$P = 30(4) + 20(0) = \$120$

Answer we get a maximum profit of \$130 with 3 trays corn muffins & 2 trays bran muffins

Suppose you make and sell skin lotion. A quart of regular skin lotion contains 2 cups oil and 1 cup of cocoa butter. A quart of extra-rich skin lotion contains 1 cup of oil and 2 cups of cocoa butter. You will make a profit of \$10/qt. on regular lotion and a profit of \$8/qt. on the extra-rich lotion. You have 24 cups of oil and 18 cups of cocoa butter. <u>How many quarts of each type of lotion should you make in order to maximize profit?</u>

let x= regular lotion y= extra rich lotion

Constraints: Table:

	Oil	cocoa butter
regular lotion	2	1
extra rich lotion	1	2
have	24	18

4.objective function: $P = 10x + 8y$

$2x + 1y \le 24$

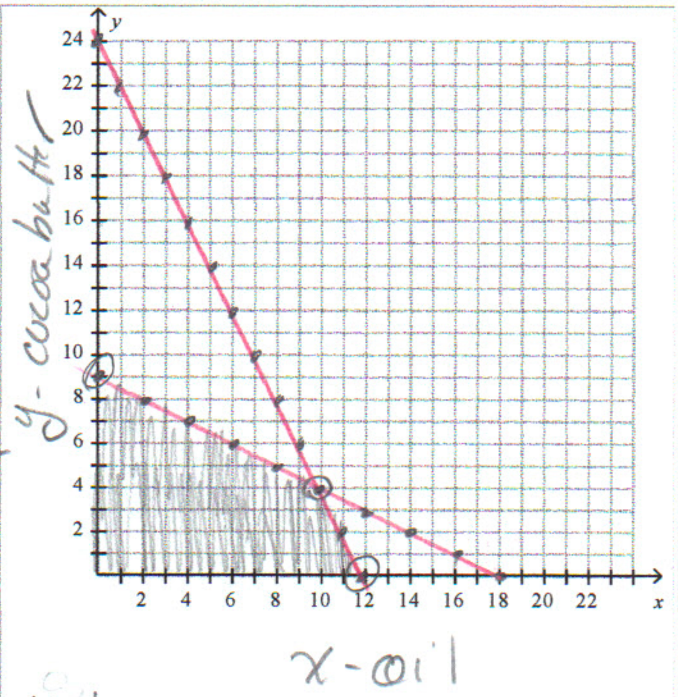
$y \le -2x + 24$

$1x + 2y \le 18$

$\frac{1}{2}(2y \le -x + 18)\frac{1}{2}$

$y \le -\frac{1}{2}x + 9$

& $y \ge 0$, $x \ge 0$



Vertices

$(0,9) \rightarrow P = 10(0) + 8(9) = \$72$

$(10,4) \rightarrow P = 10(10) + 8(4) = \$132$

$(12,0) \rightarrow P = 10(12) + 8(0) = \$120$

Max profit is \$132.00 making 10 regular & 4 extra rich lotions