6[?]

Algebra 2

Lesson 3.2 – Solving Systems Algebraically

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There are two ways to solve a system of equations algebraically (yeah, this was taught in Algebra 1!!):

- **Substitution** – substitute one equation into the other.
- **Elimination** – get rid of one variable on both equations and solve.

Substitution requires that you solve one equation in terms of one of the variables. Then whatever the variable is equal to is substituted into the second equation. The new equation is now in terms of one variable; solve for the variable. This value is then put back into one of the original equations and solve for the second variable.

Example: Solve the system of equations using substitution.

$$\begin{cases} y = x + 3 \\ 5x + y = 9 \end{cases}$$

$5x + (x+3) = 9$

$5x + x + 3 = 9$

$6x + 3 = 9$

$-3 \quad -3$

$\left(\tfrac{1}{6}\right)6x = (6)\tfrac{1}{6}$

$x = 1$

#1 $y = 1 + 3 = 4$

Check

$5(1) + y = 9$

$-5 \quad -5$

$y = 4$

Answer $(1, 4)$

$$\begin{cases} 2x + y = -1 \\ 6x - 3y = -33 \end{cases} \Rightarrow y = -2x - 1$$

$6x - 3(-2x - 1) = -33$

$6x + 6x + 3 = -33$

$12x + 3 = -33$

$-3 \quad -3$

$\left(\tfrac{1}{12}\right)12x = (-36)\tfrac{1}{12}$

$x = \frac{-36}{12} = -3$

$x = -3$

$2(-3) + y = -1$

$-6 + y = -1$

$+6 \quad +6$

$y = 5$

$6(-3) - 3(5) \stackrel{?}{=} -33$

$-18 - 15 = -33$ ✓

Ans $(-3, 5)$

$$\begin{cases} 2x - 3y = 6 \\ x + y = -12 \end{cases} \quad x = -y - 12$$

$2(-y - 12) - 3y = 6$

$-2y - 24 - 3y = 6$

$+24 \quad +24$

$\frac{-5y}{-5} = \frac{30}{-5}$

$y = -6$

$x - 6 = -12$

$+6 \quad +6$

$x = -6$

Ans $(-6, -6)$

Elimination requires that you line up "like" terms from each equation and "eliminate" one variable by adding the two equations together, solve the resulting equation and substitute the answer into either equation to find the value into either equation to find the value of the second variable. **Note** you may need to take one equation and make an equivalent equation. When the two equations are added together a variable cancels out.

Example: Solve the system of equations using elimination.

ADD

$$\begin{cases} 3x + y = -9 \\ -3x - 2y = 12 \end{cases}$$

$-y = 3$

$\boxed{y = -3}$

$3x - 3 = -9$
$+3 \quad +3$

$3x = -6$

$x = -2$

Ans $(-2, -3)$

$-3(-2) - 2(-3) \stackrel{?}{=} 12$

$6 + 6$

$$\begin{aligned} 2\,(4x + 3y &= -6)\,2 \\ 5x - 6y &= -27 \end{aligned}$$

$8x + \cancel{6y} = -12$

$\frac{13x}{13} = \frac{-39}{13}$

$x = -3$

$13 \times 3 = 39$

$4(-3) + 3y = -6$

$-12 + 3y = -6$
$+12 \quad +12$

$\frac{3y}{3} = \frac{6}{3}$

$y = 2$

Ans $(-3, 2)$

$$\begin{aligned} 2\,(3x + 7y &= 15)\,(2) \\ -7\,(5x + 2y &= -4)\,-7 \end{aligned}$$

$6x + \cancel{14y} = 30$

$-35x - \cancel{14y} = 28$

$\frac{-29x}{-29} = \frac{58}{-29}$

$x = -2$

Solve for y

$$\begin{aligned} 2\,(2x - y &= 3) \\ -4x + 2y &= -6 \end{aligned}$$

$4x - 2y = 6$

$0 = 0$

True

same line

Answer is all real solutions

Dependent lines

$$\begin{cases} 2x - 3y = 18 \\ -2x + 3y = -6 \end{cases}$$

$0 = 12$

False – (Inconsistent lines)

no solutions (parallel lines)

Remember to check your answers. A graphing calculator can verify your algebraic process by showing you the point of intersection. In summary, the steps for:

SUBSTITUTION

1. Solve one equation for one-variable
2. substitute that equation into the other equation
3. solve for the variable
4. solve for the remaining variable
5. check answers

ELIMINATION

1. line up variables
2. eliminate one variable by adding the equations
3. solve resulting equation for the variable
4. solve for the other variable
5. check answers.

Which method is the best for solving the following systems. Choose from graphing, substitution, or elimination. Don't solve, unless you want to. Why did you choose the system you chose?

$\begin{cases} 3x - y = 5 \\ y = 4x + 2 \end{cases}$ Substitute in for y of equ 1	$\begin{cases} 2x + 3y = 4 \\ 2x + 5y = -6 \end{cases}$ multiply one equ by -1 to be ready for elimination	$\begin{cases} y = 3x - 5 \\ y = \cancel{4x2+} \end{cases}$ 4x+2 3x-5=4x+2 Substitution Substitute

3x-5 for "y" of equ 2.