

Algebra 2

Lesson 3.2 – Solving Systems Algebraically

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There are two ways to solve a system of equations algebraically (yeah, this was taught in Algebra 1!!):

- **Substitution** – substitute one equation into the other.
- **Elimination** – get rid of one variable on both equations and solve.

Substitution requires that you solve one equation in terms of one of the variables. Then whatever the variable is equal to is substituted into the second equation. The new equation is now in terms of one variable; solve for the variable. This value is then put back into one of the original equations and solve for the second variable.

Example: Solve the system of equations using substitution.

$\begin{aligned} x + 3y &= 7 \\ 2x - 4y &= 24 \end{aligned}$	<p>solve 1st equ. for x: $x + 3y - 3y = 7 - 3y$ $x = 7 - 3y$</p> <p>substitute into 2nd equ. $2x - 4y = 24$ $2(7 - 3y) - 4y = 24$ distribute and combine like terms: $14 - 6y - 4y = 24$ $-10y = 10$ $y = -1$</p>	<p>plug y into equation and solve for x: $x + 3(-1) = 7$ $x - 3 = 7$ $x = 10$</p> <p>the solution $(x, y) = (10, -1)$</p> <p>check: $2(10) - 4(-1) = 24$ $20 + 4 = 24 \checkmark$</p>
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Elimination requires that you line up “like” terms from each equation and “eliminate” one variable by adding the two equations together, solve the resulting equation and substitute the answer into either equation to find the value into either equation to find the value of the second variable. **Note** you may need to take one equation and make an equivalent equation so that when the two equations are added together a variable cancels out.

Example: Solve the system of equations using elimination.

$\begin{aligned} 3x + 2y &= 10 \\ x + y &= 6 \end{aligned}$ <p><i>multiply equation by -2. Then when you add the equations the y's will fall out</i></p>	<p>multiply by -2 $3x + 2y = 10$ $-2x - 2y = -12$ add the equations $x = -2$</p> <p>the solution $(x, y) = (-2, 8)$</p>	<p>plug $x = -2$ into one of the equations and solve for y: $-2 + y = 6$ $y = 8$</p> <p>check: $3(-2) + 2(8) = 10$ $-6 + 16 = 10$ $10 = 10 \checkmark$</p>
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Remember to check your answers. A graphing calculator can verify your algebraic process by showing you the point of intersection. In summary, the steps for:

SUBSTITUTION	ELIMINATION
<ol style="list-style-type: none"> 1. Solve one equation for one-variable 2. substitute that equation into the other equation 3. solve for the variable 4. solve for the remaining variable 5. check answers 	<ol style="list-style-type: none"> 1. line up variables 2. eliminate one variable by adding the equations 3. solve resulting equation for the variable 4. solve for the other variable 5. check answers.

Example -

Substitution - Substitute 1 equ. into the other:

* $\begin{cases} y = x + 3 \\ 5x + y = 9 \end{cases}$ Substitute into equation #2 $\Rightarrow 5x + x + 3 = 9$ $6x + 3 = 9$ $\frac{1}{6} 6x = 6 \frac{1}{6} 6 = 1$ $x = 1$
 $y = 1 + 3 = 4$ Check $5(1) + 4 = 9$ $9 = 9 \checkmark$
 $(1, 4)$

* $\begin{cases} 2x + y = -1 \\ 6x - 3y = -33 \end{cases} \Rightarrow y = -2x - 1$ $6x - 3(-2x - 1) = -33$ $6x + 6x + 3 = -33$ $(\frac{1}{12}) 12x = -36(\frac{1}{12})$ $x = -3$
 $y = -2(-3) - 1 = 5$ Check: $6(-3) - 3(5) = -33$ $-18 - 15 = -33 \checkmark$
 $(-3, 5)$

* $\begin{cases} 2x - 3y = 6 \\ x + y = -12 \end{cases}$ $2(-12 - y) - 3y = 6$ $-24 - 2y - 3y = 6$ $-5y = 30$ $y = -6$ $x = -12 - y = -12 - (-6) = -6$ Check: $2(-6) - 3(-6) = 6$ $-12 + 18 = 6 \checkmark$
 $(-6, -6)$

Elimination - get rid of a variable:

$\begin{cases} 3x + y = -9 \\ -3x + 2y = 12 \end{cases}$ ADD $3y = 3$ $y = 1$ $3x + 1 = -9$ $3x = -10$ $x = -\frac{10}{3}$ Check $3x + 2 = 12$ $-3x = 10$ $\checkmark x = -\frac{10}{3}$ Ans $(-\frac{10}{3}, 1)$

2 $(4x + 3y) = (-6) \cdot 2 \rightarrow 8x + 6y = -12$ $5x - 6y = -27$ $13x = -39$ $x = -3$ $4(-3) + 3y = -6$ $-12 + 3y = -6$ $3y = 6$ $y = 2$ Check $5(-3) - 6(2) = -27$ $-15 - 12 = -27 \checkmark$
 $(-3, 2)$

Multiply both equations:

$$\begin{aligned} (2)(3x+7y) &= (15)(2) \rightarrow 6x+14y=30 \\ (-1)(5x+2y) &= (-1)(-1) \rightarrow -5x-2y=1 \end{aligned}$$

$$\boxed{(-2, 3)}$$

$$\begin{aligned} 6x+14y &= 30 \\ -5x-2y &= 1 \\ \hline -29x &= 58 \\ x &= -2 \end{aligned}$$

$$\begin{aligned} 3(-2)+7y &= 15 \quad \text{check:} \\ -6+7y &= 15 \\ 7y &= 21 \\ y &= 3 \end{aligned}$$

Multiply legs

$$\begin{aligned} 2(2x-y) &= (3)(2) \rightarrow 2x-2y=6 \\ -4x+2y &= -6 \rightarrow -4x+2y=-6 \end{aligned}$$

$$\boxed{(0, -3)}$$

$$\begin{aligned} 2x-2y &= 6 \\ -4x+2y &= -6 \\ \hline -2x &= 0 \\ x &= 0 \end{aligned}$$

$$\begin{aligned} 2(0)-y &= 3 \quad \text{check} \\ -y &= 3 \\ y &= -3 \end{aligned}$$

Best Method?

$$3x-y=5$$

$$y=4x+2$$

substitution \rightarrow

$$\boxed{(-7, -26)}$$

$$3x-(4x+2)=5$$

$$3x-4x-2=5$$

$$-x=7$$

$$x=-7$$

$$y=4(-7)+2$$

$$y=-28+2$$

$$y=-26$$

check

$$3(-7)-(-26)=5$$

$$-21+26=5 \checkmark$$

check:

$$(-1)(2x+3y)=(4)(-1)$$

$$2x+3y=4$$

ready for elimination

$$\boxed{(2, 0)}$$

$$-2x-3y=-4$$

$$2x-5y=4$$

$$-8y=0$$

$$y=0$$

$$2x+3(0)=4$$

$$2x=4$$

$$x=2$$

$$2(2)-5(0)=4$$

$$4=4 \checkmark$$

$$y=3x-5$$

$$y=4x+2$$

substitution

$$\boxed{(-7, -26)}$$

$$\begin{aligned} -3x-5 &= 4x+2 \\ -7 &= x \end{aligned}$$

$$y=3(-7)-5$$

$$y=-21-5$$

$$y=-26$$

$$\text{check: } -26 \stackrel{?}{=} 4(-7)+2$$

$$-26 = -28+2 \checkmark$$