Algebra2
Lesson 2-2: Linear Equations
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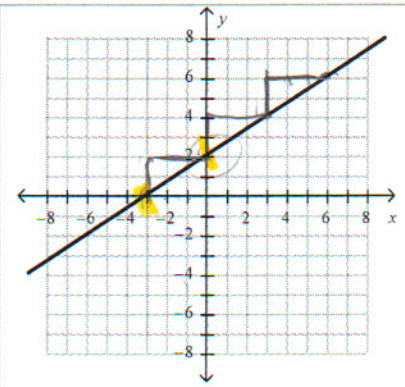
In Algebra I we discovered that a function is considered to be linear if the independent variable increases or decreases at a constant rate. Graphically, a function that graphs out to be a line is a linear function.

Vocabulary
x – This is the independent variable and is graphed on the horizontal axis.
y – This is the dependent variable and is the output of the function resulting when an x-value is put into the linear equation.
x-intercept – The location where a line crosses the x-axis.
y-intercept – The location where a line crosses the y-axis.

y-intercept has the form (0,y) x-intercept has the form (x,0)

Example:

state the x and y intercepts for the graph at the right.



(-3, 0) ← x-intercept
(0, 2) = y-intercept

rate of change
$\frac{\Delta y}{\Delta x} = \frac{2}{3}$

rate of change – the measure of the steepness of the line. It is ratio of the vertical change over the horizontal change between two points. The rate of change is also called the **slope**:

$$rate\ of\ change = \boldsymbol{slope} = \boldsymbol{m} = \frac{change\ in\ f(x)}{change\ in\ x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\boldsymbol{rise}}{\boldsymbol{run}} = \frac{\boldsymbol{y_2 - y_1}}{\boldsymbol{x_2 - x_1}} = \frac{\Delta y}{\Delta x}$$

Delta

$m = \frac{y_2 - y_1}{x_2 - x_1}$ ← slope formula

Example: Does the xy table have a constant rate of change? Does the table model a linear function? Check all the points!

Constant Rate of change ⇒ linear

x	0	2	4	6	8
f(x)	-2	0	6	16	30

+2 +2 +2 +2

$\frac{y}{x}$ $\frac{2}{2}$ +2 +6

* no const. rate of chng.
* not linear

x	0	2	3	5	6
y	3	7	9	13	15

+2 +1 +2 +1

+4 +2 +4 2

$\frac{y}{x}$ $\frac{4}{2}=2$ $\frac{2}{1}$ $\frac{4}{2}=2$ $\frac{2}{1}$

① yes const.
② linear

What is the slope of a line passing through the points $(0, -3)$ *and* $(7, -9)$

(x_1, y_1) (x_2, y_2)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-9 - (-3)}{7 - 0} = \frac{-6}{7} = \text{slope}$

Standard Form $\Rightarrow Ax + By = C$

Point-Slope Formula

A line passing through point (x_1, y_1) with a slope m has the equation: point.

Variables (x, y) $\quad y - y_1 = m(x - x_1)$

$\frac{y_2 - y_1}{x_2 - x_1} = m$

Write in standard form the equation of each line:

slope 2, through $(4, -2)$ $\quad x_1, y_1$

$y - y_1 = m(x - x_1)$

$y - (-2) = 2(x - 4)$ pt slope fm

$y + 2 = 2x - 8$

$+2 \quad +8$

$y + 10 = 2x$

$-y \quad -y$

$\Rightarrow 2x - y = 10$

Write in point slope form the equation of the line that passes through the points $(5, 1)$ and $(-4, -3)$

$m = \frac{-3 - 1}{-4 - 5} = \frac{-4}{-9} = m = 4/9$ $\quad x_1 y_1 \quad x_2 y_2$

$y - 1 = \frac{4}{9}(x - 5)$ OR

$y - (-3) = \frac{4}{9}(x - (-4))$ ← simplify

$y + 3 = \frac{4}{9}(x + 4)$

Graphing given a point and a slope

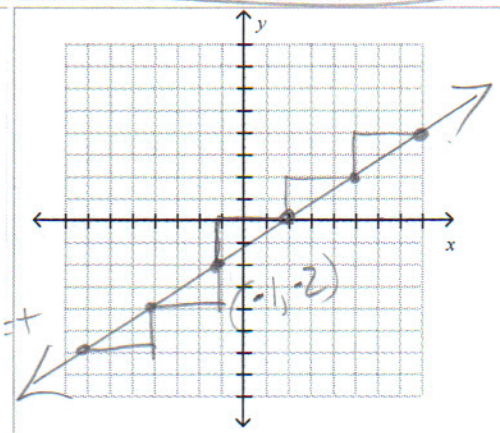
A slope makes graphing a line from a given point very easy.

Graph a line through (-1,-2) with a slope of $\frac{2}{3}$.

(8,4)

$\frac{\uparrow 2}{3\rightarrow} \; \frac{+}{+}$

$\frac{\downarrow 2}{\leftarrow 3} \; \frac{-}{-} = +$



Slope-Intercept Form

Combining the ideas about slope and intercept lead to a general equation form for a line called the slope-intercept form: y=mx+b, where m is the slope and b is the y-intercept. The slope-intercept form allows one to graph almost any linear equation in just a few seconds WITHOUT the use of a graphing calculator

y-intercept

$y = mx + b$

slope

Find the slope using slope-intercept form:

$3x + 2y = 1$ $\quad -3x \quad -3x$

$\frac{1}{2}(2y) = (-3x + 1)\frac{1}{2}$

$y = \frac{-3}{2}x + \frac{1}{2}$

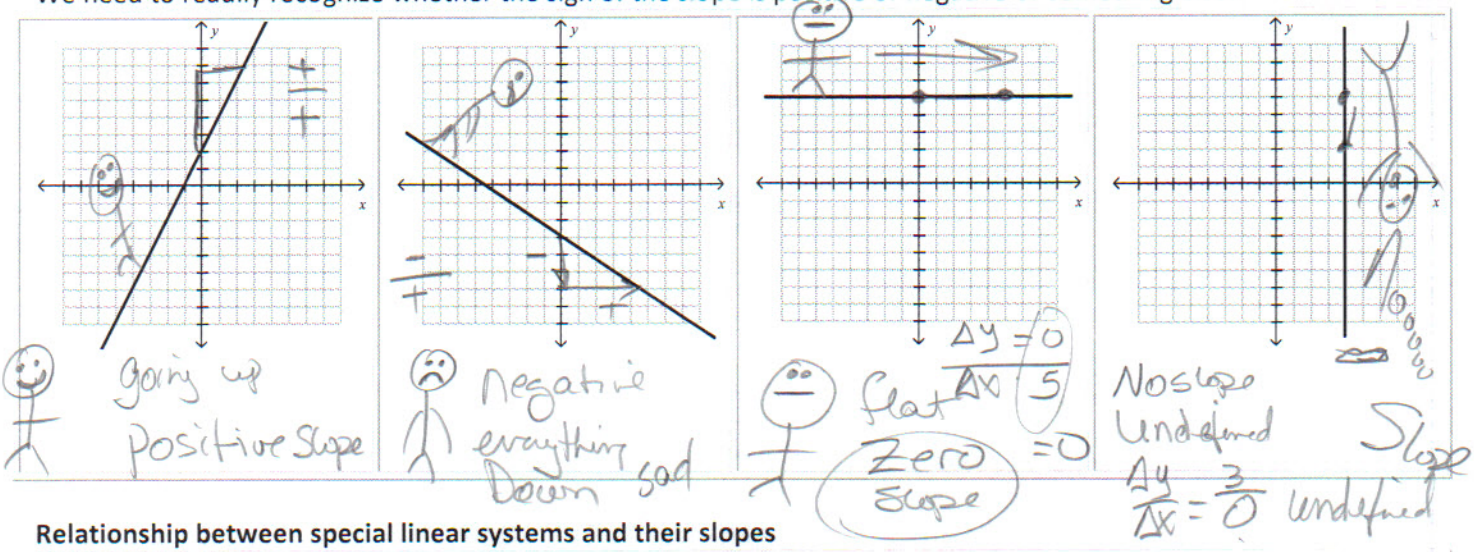
$Ax + By = C$ $\quad -Ax \quad -Ax$

$\frac{1}{B}(By) = (-Ax + C)\frac{1}{B}$

$y = \frac{-A}{B}x + \frac{C}{B}$

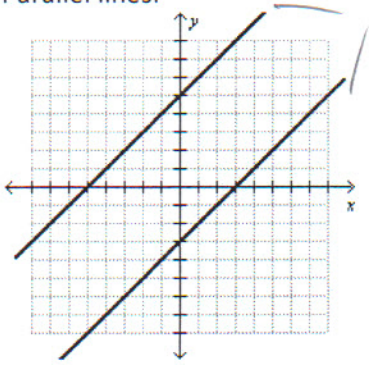
$m = \frac{-A}{B} = \frac{-3}{2} \quad b = \frac{C}{B} = \frac{1}{2}$

We need to readily recognize whether the sign of the slope is positive or negative or something else!



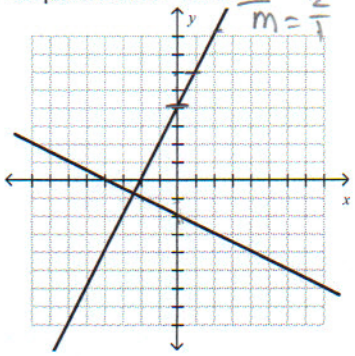
Relationship between special linear systems and their slopes

Parallel lines:



Slope is same for parallel lines
$m =$
$b \neq$

Perpendicular lines:



⊥ slopes opposite +/− reciprocal

$m = \frac{2}{1}$

$\perp -\frac{1}{2} = m\perp$

We are able to find an equation of a line passing through a point and perpendicular to another line if we are given the reference line and the point. Also, we can find a line parallel to another if given the same information.

$(-1, 3)$ *and perpendicular to the line* $y = 5x - 3$

$m = 5$

$\perp m = -\frac{1}{5}$

- recip 1 $-\frac{1}{5}$

$y - 3 = \frac{-1}{5}(x + 1)$

perpendicular to $y = 5x - 3$ & pass through $(-1, 3)$

$(2, 1)$ *and parallel to the line* $y = \frac{2}{3}x + \frac{5}{8}$

Slopes Same

$m = \frac{2}{3}$

$y - 1 = \frac{2}{3}(x - 2)$