Algebra II
Lesson 10-6: Translating Conic Sections
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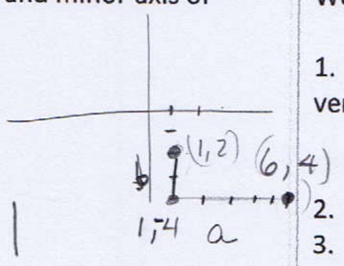
We have seen with our conics sections that they are not limited to being centered about the vertex, but may be translated about the x – y planes; the center is at (h, k). The table below summarizes the equations for the conics sections both centered at the origin and how a translation will alter the standard form:

Conics Section	Center/vertex at $(0,0)$	Center/Vertex at (h,k)	Foci
Circle (10.3)	Center $(0,0)$ $x^2 + y^2 = r^2$	Center (h,k) $(x-h)^2 + (y-k)^2 = r^2$	NA
Ellipse (10.4) $\boldsymbol{a^2 - b^2 = c^2}$	Center $(0,0)$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	Center (h,k) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	$(h \pm c, k)$ $(h, k \pm c)$
Hyperbola (10.5) $\boldsymbol{a^2 + b^2 = c^2}$	Center $(0,0)$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	Center (h,k) $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	$(h \pm c, k)$ $(h, k \pm c)$
Parabola (10.2) $\boldsymbol{\lvert a \rvert = \frac{1}{4c}}$	Vertex $(0,0)$ $y = ax^2$ $x = ay^2$	Vertex (h,k) $y = a(x-h)^2 + k$ $x = a(y-k)^2 + h$	$(h, k \pm c)$ $(h \pm c, k)$

Let's do a review of our translation problems:

Write an equation of an ellipse with a center of $\underset{(h\ k)}{(1, -4)}$, horizontal major axis of length 10 and minor axis of length 4	We need to understand some terminology first.
$a = 5$ $b = 2$ $\frac{(x-1)^2}{25} + \frac{(y+4)^2}{4} = 1$ 	1. Length of the major axis: that is from vertex to vertex or **2a**, we need just **a** 2. Identify the values of (h,k) 3. Which is the major axis? 4. Plug into the standard formula.

Write the equation of a hyperbola with vertices (2,-1) and (2,7) and foci (2,10) and (2,-4).	1. Sketch the points so you know the orientation of the hyperbola. Vertices show a vertical orientation.

(h,k) midpoint $\frac{7+(-1)}{2} = \frac{6}{2} = 3$

$(2,3)$

$a = 7-3 = 4 \qquad a^2 = 16$

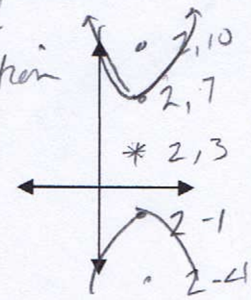
$c = 10-3 = 7 \qquad c^2 = 49$

$16 + b^2 = 49$

$b^2 = 33$

$$\frac{(y-3)^2}{16} - \frac{(x-2)^2}{33} = 1$$

Vertical orientation



2. Where is the midpoint of a line through the vertices? Use the midpoint formula to find *y*.

3. What is the foci? With a and c what is b?

Identify the conic section with the following equation: $x^2 + y^2 - 12x + 4y = 8$

x & y squared w/ same coefficient ∴ circle

$x^2 - 12x + 36 + y^2 + 4y + 4 = 8 + 36 + 4$

$(x-6)^2 + (y+2)^2 = 48$

circle $(h,k) = (6,-2)$

radius $= \sqrt{48}$

1. Group like terms together (x and y) and locate the constant on the right side of the equation

2. Complete the square ½ of the linear term coefficient squared, add to both sides of the equation

3. If we divide through by the constant to get a 1 for the standard form of an ellipse we will see that the axes are equal, hence we have a **circle** with a radius of $\sqrt{48}$

Try: $4x^2 + 9y^2 + 16x - 54y = -61$

x & y squared coeff. different ∴ ellipse

$4x^2 + 16x$

$4x^2 + 16x + 16 + 9y^2 - 54y + 81 = -61 + 16 + 81$

$4(x^2 + 4x + 4) + 9(y^2 - 6y + 9)$

$$\frac{4(x+2)^2}{36} + \frac{9(y-3)^2}{36} = \frac{36}{36}$$

$$\frac{(x+2)^2}{9} + \frac{(y-3)^2}{4} = 1$$

center $(-2,3)$

foci $(-2 \pm \sqrt{52}, 3)$

After we group like terms together and set about to complete the square, we need to understand how to handle a coefficient in front of the quadratic term.

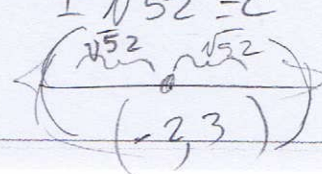
1. Factor the leading coefficient out, if at all possible. **Square ½ the linear coefficient and then multiply that product with outside factor.** For x, ½ of 4 squared is 4 multiply by the outside 4. What is added to the left, add to the right…

2. follow through and simplify

3. What is the conic?

$36 + 16 = 52$

$\pm\sqrt{52} = c$



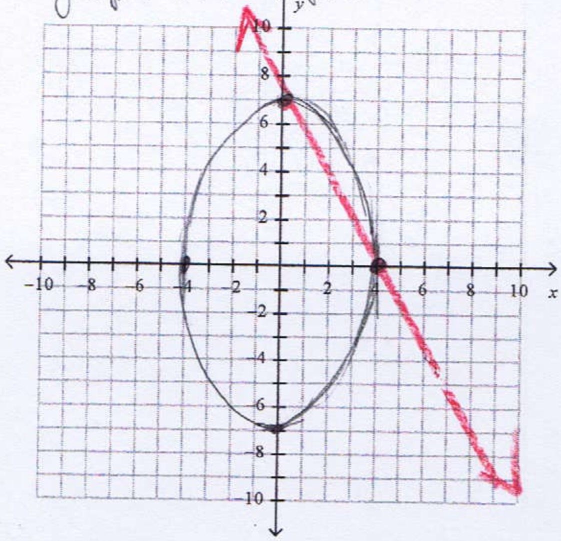
In Algebra I and in the 1st semester of algebra II we calculated the intersection of two lines; i.e. solving a system of equations. Well, system of equations may involve conic equations; they are not reserved for linear equations.

Solve the system of equations by graphing:

$$\begin{cases} \dfrac{x^2}{16} + \dfrac{y^2}{49} \\ 7x + 4y = 28 \end{cases}$$

ellipse $a = 7$, $b = 4$ ↕ vertical

graph intercepts



(0,7)

(4,0)

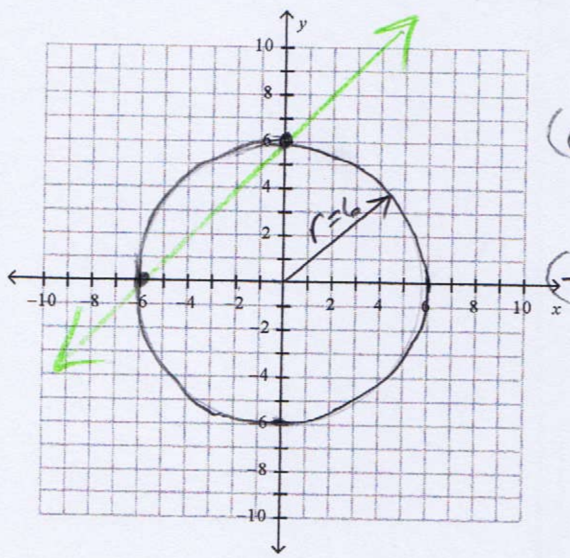
Where do we see the line & circle intersect?

Solution

(4,0) & 0,7

$$\begin{cases} x^2 + y^2 = 36 \\ -x + y = 6 \end{cases}$$

Circle $r = 6$ center(0,0)



(0,6)

(−6,0)

Solution at intersection:

Solution:

(0,6)

(−6,0)