

Algebra II
Lesson 10.4 Ellipses
Mrs. Snow Instructor

Ellipses a.k.a. "ovals" are best known in their role in science. The orbits of the planets are elliptical about the sun. In acoustics an elliptical shape room will allow sound to reflect and be easily heard from some distance, hence the vaulted ceilings of our capital, cathedrals and many other places. In mechanics, gears may be elliptical in shape to allow for changes in torque. This is to name only a few of many places ellipses are found.

By definition: An ellipse is a set of points P in a plane such that the sum of the distances from P to two fixed points F_1 and F_2 is a given constant k .



$$PF_1 + PF_2 = k, \text{ where } k > F_1F_2$$

foci: The fixed points, F_1 and F_2 , are called (singular is **focus**)

major axis: the longest axis and contains the foci.

vertices of the ellipse: end points of the major axis that lie on the ellipse itself

minor axis: the shorter of the two axes and is perpendicular to the major axis

co-vertices: end points of the minor axis.

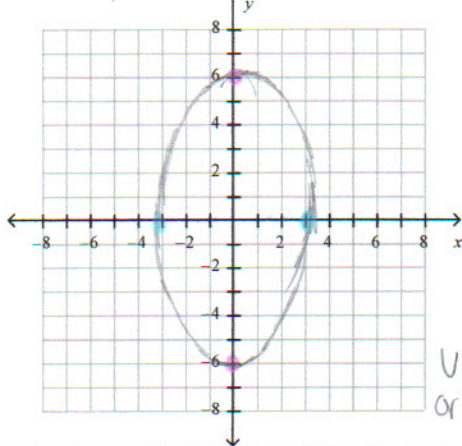
The standard form of an equation of an ellipse with a center at the origin and where $a > b > 0$	
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ largest denominator major axis here the major axis is horizontal and equals $2a$ in length vertices: $(\pm a, 0)$ co-vertices: $(0, \pm b)$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ largest denominator major axis the major axis is vertical and equals $2a$ in length vertices: $(0, \pm a)$ co-vertices: $(\pm b, 0)$

Writing equations when centered about the origin and graph:

vertex of $(0, -6)$ and co-vertex at $(3, 0)$.

$$\frac{x^2}{3^2} + \frac{y^2}{6^2} = 1 \Rightarrow \frac{x^2}{9} + \frac{y^2}{36} = 1$$

$$a = 6 \quad b = 3$$

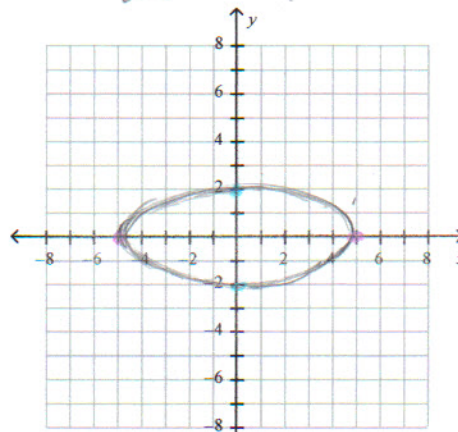


Vertical orientation

vertex at $(5, 0)$ and co-vertex at $(2, 0)$

$$a = 5 \quad b = 2$$

$$\frac{x^2}{5^2} + \frac{y^2}{2^2} = 1$$



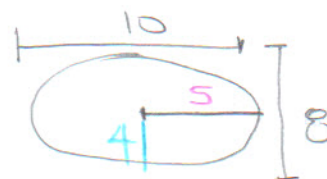
Horizontal orientation

Knowing the length of each axis and the center of the ellipse at the origin you can also come up with the equation. *always draw a rough sketch to help you see what is going on!!*

Find the equation of an ellipse given the horizontal axis has a length of 10 units and the vertical axis has a length of 8 units, and is centered at the origin.

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

↑
Note: largest denominator: horizontal orientation



$$a = 5$$

$$b = 4$$

Finding the foci:

The foci are important points in an ellipse. The sun is a focus point in the elliptical orbit of the earth. *The foci are always on the major axis at c units from the center.*

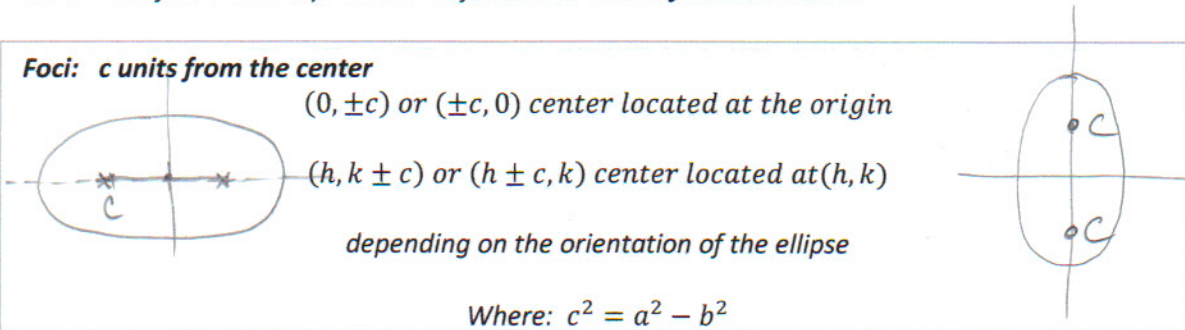
Foci: c units from the center

$(0, \pm c)$ or $(\pm c, 0)$ center located at the origin

$(h, k \pm c)$ or $(h \pm c, k)$ center located at (h, k)

depending on the orientation of the ellipse

$$\text{Where: } c^2 = a^2 - b^2$$



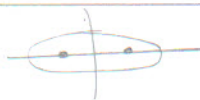
STEPS:

1. Write in standard form
2. Largest denominator indicates whether the x or y -axis is the major axis.
3. Use the equation for finding the foci, $c^2: a^2 - b^2 = c^2$

4. Solve for c and write the foci as ordered pairs.

Find the foci

$$c^2 = a^2 - b^2$$



$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

$$a = 5 \quad b = 2$$

$$c^2 = 25 - 4$$

$$c^2 = 21 \quad c = \pm\sqrt{21}$$

Foci $(\sqrt{21}, 0)$ $(-\sqrt{21}, 0)$

$$\frac{1}{225}(25x^2 + 9y^2) = 225\left(\frac{1}{225}\right)$$

ellipse = 1

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

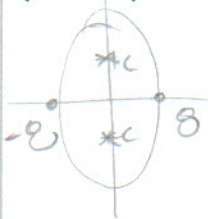


$$c^2 = 25 - 9 = 16$$

$$c = \pm 4$$

foci $(0, 4)$ $(0, -4)$

Write an equation for an ellipse: Foci at $(0, \pm\sqrt{17})$ and co-vertices at $(\pm 8, 0)$



Vertical orientation

$$\frac{x^2}{8^2} + \frac{y^2}{?} = 1$$

$$c^2 = a^2 - b^2$$

$$c = \sqrt{17}$$

$$c^2 = 17$$

$$17 = a^2 - 64$$

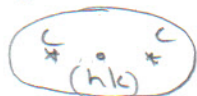
$$81 = a^2$$

$$9 = a$$

$$\Rightarrow \frac{x^2}{64} + \frac{y^2}{81} = 1$$

1. Rough sketch will help!
2. ID whether you have a vertex or co-vertex.
3. Use $a^2 - b^2 = c^2$ to find missing variable.

Ellipses NOT centered at the origin:

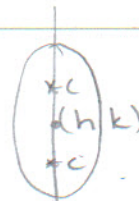


$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \text{ or } \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Center is located at (h, k)

foci $(h \pm c, k)$

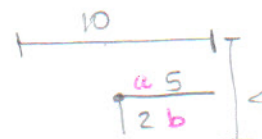
$(h, k \pm c) = \text{foci}$



Write the equation of an ellipse with a center $(1, -4)$, horizontal major axis of length 10, and minor axis of length 4.

length = 4 $b = 2$

$$\frac{(x-1)^2}{25} + \frac{(y+4)^2}{4} = 1$$



Completing the square:

When the leading coefficient is >1 , factor out the coefficients

Find the center and foci

$$2x^2 + 8x + y^2 + 4 = 0$$

$$2(x^2 + 4x + 4) + y^2 = -4 + 8$$

$$\frac{1}{4}(2(x+2)^2 + y^2) = (4) \frac{1}{4}$$

ellipses = 1 & denominators

$$\frac{(x+2)^2}{2} + \frac{y^2}{4} = 1$$

$$\frac{(x+2)^2}{2} + \frac{y^2}{4} = 1 \text{ Ans}$$

$(-2, 0)$ Center

$$c^2 = a^2 - b^2$$

$$c^2 = 4 - 2 = 2$$

$$c = \pm\sqrt{2}$$

$\Rightarrow$ vertical orientation



$(-2, \pm\sqrt{2})$ foci

Find the center and foci

$$25x^2 + 16y^2 + 150x = 160y - 225$$

$$25x^2 + 150x + 16y^2 + 160y = -225$$

$$25(x^2 + 6x + 9) + 16(y^2 + 10y + 25) = -225 + 225 + 400$$

$$\frac{25(x+3)^2}{400} + \frac{16(y+5)^2}{400} = \frac{400}{400}$$

$$\frac{(x+3)^2}{16} + \frac{(y+5)^2}{25} = 1 \text{ Ans}$$

$(-3, -5)$ center

$$c^2 = 25 - 16 = 9; c = \pm 3$$

Vertical orientation

foci



$$(-3, -5+3) = (-3, -2)$$

$$(-3, -5-3) = (-3, -8)$$