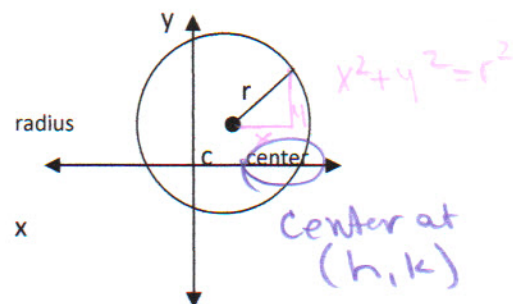


Algebra II
Lesson 10-3: Circles
Mrs. Snow, Instructor

A circle is a set of points in a plane that are a distance r or radius from a fixed point (called center with coordinates (h,k)).

A circle's formula is derived from the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Our distance is in fact the radius. Squaring both sides and setting the center at the origin, $(0,0)$, we get: $x^2 + y^2 = r^2$. When the circle is not centered at the origin the equation takes on the form: $(x - h)^2 + (y - k)^2 = r^2$, where the center is at the point, (h,k) .



Circles have a myriad of applications like computing lumber volume of trees, finding the epicenter of earthquakes, optimizing gear sizes and ratios for maximum engine horse power. If you want the best deal, take a ratio of area to price to see which pizza really is the best price!

Standard form, with center at $(0, 0)$

$$x^2 + y^2 = r^2$$

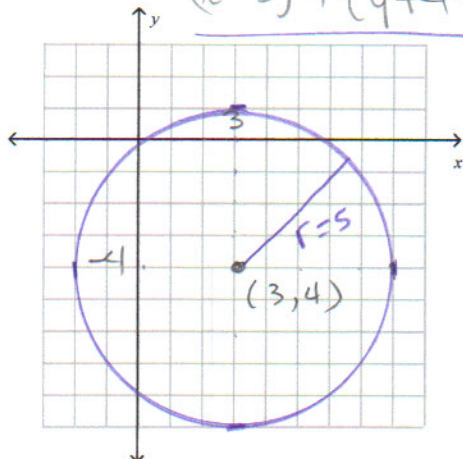
Standard form with center at (h, k)

$$(x - h)^2 + (y - k)^2 = r^2$$

Writing an equation of a circle: Given a center and radius write the equation of a circle and graph

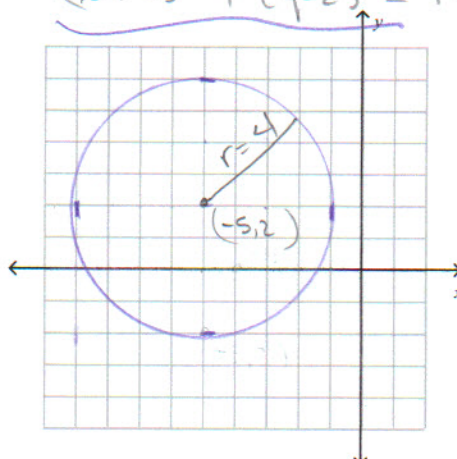
Center $(3, -4)$ radius of 5

(h, k) $(x - 3)^2 + (y - (-4))^2 = 5^2$
 $(x - 3)^2 + (y + 4)^2 = 25$

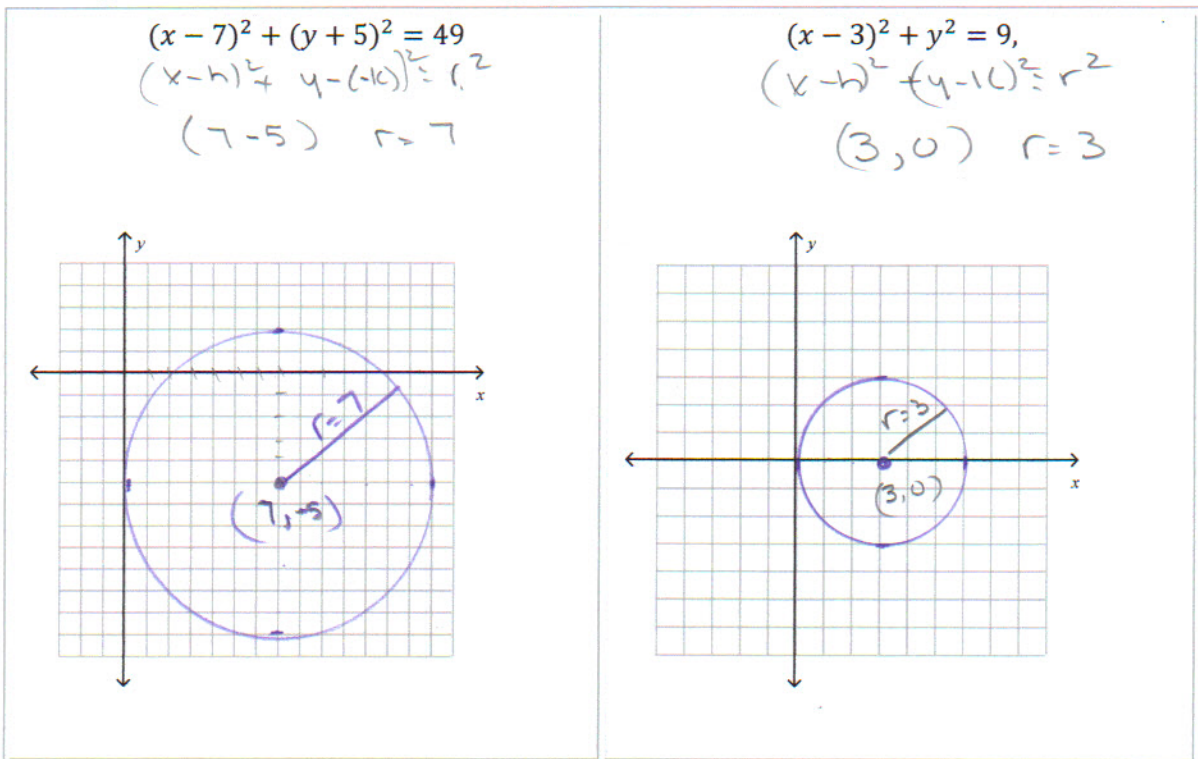


Center $(-5, 2)$ radius of 4

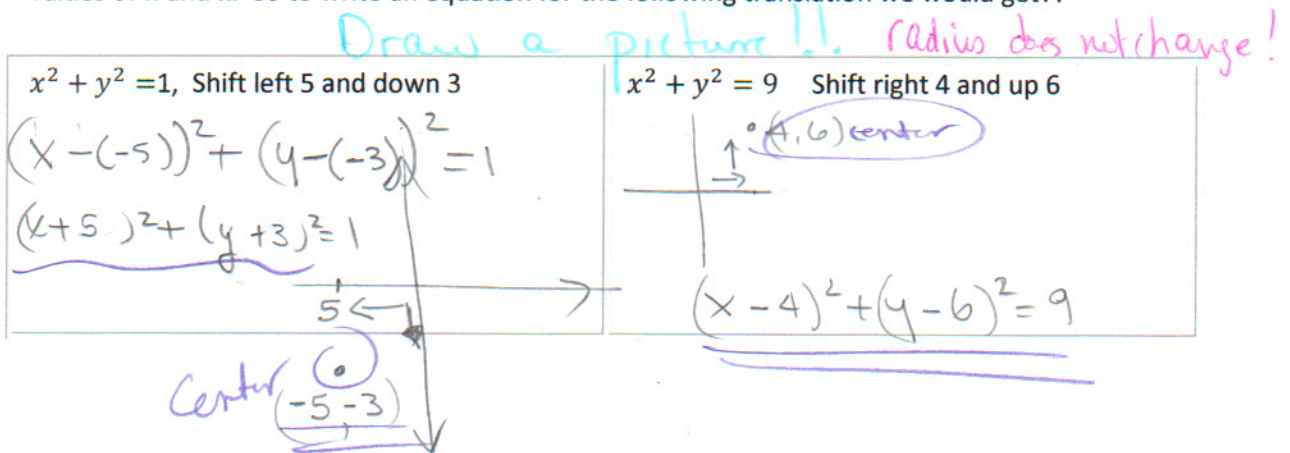
(h, k) $(x - (-5))^2 + (y - 2)^2 = 4^2$
 $(x + 5)^2 + (y - 2)^2 = 16$



Finding the center of a circle, radius and graph:

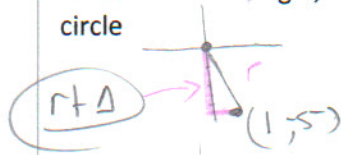


As with all our other equations involving the point, (h, k) , these values may be used in conjunction with the standard form of a circle to perform translations. Here we look at the values of h and k . So to write an equation for the following translation we would get??



You can also find the equation to a circle when given the center and a point on the circumference. How do you suppose you can do this??? (Hint: To write an equation of a circle you need two things, the center and the radius.)

Given a point on a circle of (1,-5) and the center is at the origin, find the equation of the circle



$$x^2 + y^2 = r^2$$

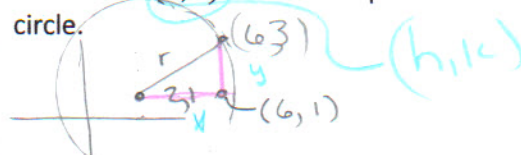
$$1^2 + (-5)^2 = r^2$$

$$1 + 25 = r^2$$

$$26 = r^2 \Rightarrow x^2 + y^2 = 26$$

note $r = \sqrt{26}$

Given a point on a circle of (6,3) and the center is at (2,1). Find the equation of the circle.



$$x = 6 - 2 = 4$$

$$y = 3 - 1 = 2$$

$$x^2 + y^2 = r^2$$

$$4^2 + 2^2 = r^2$$

$$16 + 4 = 20 = r^2$$

$$(r = 4\sqrt{5})$$

$$(x-2)^2 + (y-1)^2 = 20$$

10.6 Once upon a time in the land of completing the square.....

Write the following circle equations in standard form (refer to 5.7 for completing the square).

ID the center and radius. Graph.

$$x^2 + 2x + 1 + y^2 = 4$$

$$(x^2 + 2x + 1) + y^2 = 4 - 1 + 1$$

$$(x+1)^2 + y^2 = 4$$

center $(-1, 0)$

$r = 2$



1. YIKES!
2. Move the integers to the right side
3. Group like terms together.
4. If the variable is squared and no linear term it is finished and we can go on to the other terms.
5. Leading coefficient? Factor it out
6. Complete the square: $\frac{1}{2}$ of the linear term squared.
- 7 What we add to the left we must add to the right. Don't forget the coefficient!
8. If there is a coefficient, then multiply through to get the true value of what you added to the left side and add that amount to the right.

$$x^2 + y^2 - 6x + 4y + 4 = 0$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 = -4 + 9 + 4$$

$\frac{1}{2}(-6) \rightarrow (-3)^2$ $\frac{1}{2}(4) \rightarrow 2^2$

$$(x-3)^2 + (y+2)^2 = 9 \quad \text{Ans}$$

$(3, -2)$ center

$r = 3$ radius