Data transferred over the internet is encoded or encrypted so that someone attempting to illegally access the data will find something that is unintelligible. One way to encrypt messages and data uses matrices and their inverses. We will be looking at matrix inverses during the next lesson. Today, we are looking at determinants which are used in calculating inverses.

The determinant, abbreviated det and symbolized with \( | \) \( A \), it is a nonzero quantity (when the det=0 we have another situation that we will look at in the next lesson). For a 2×2 matrix, its determinant is found by subtracting the products of its diagonals:

Given a matrix \( A= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \), where a, b, c, and d are real numbers

\[
\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc
\]

Example: Compute the determinant of \( A= \begin{bmatrix} -3 & 4 \\ 2 & -5 \end{bmatrix} \)

\[
\det A = \begin{vmatrix} -3 & 4 \\ 2 & -5 \end{vmatrix} = (-3)(-5) - (4)(2) = 15 - 8 = 7
\]

One can also compute a determinant using a graphing calculator:

- Press \( \text{MATRIX} \) \( \rightarrow \) to \( \text{EDIT} \). Down to \( 1:[A] \). \( \text{ENTER} \)
- Enter the matrix dimensions: # rows \( \text{ENTER} \) # columns \( \text{ENTER} \). Enter the data for the 2x2 matrix in the matrix.
- Press \( 2^{\text{nd}} \) \( \text{MODE} \) \( \text{(QUIT)} \)
- Press \( \text{MATRIX} \) again. Go right once to \( \text{MATH} \). Down to \( 1:\text{det} \).
- Press \( \text{MATRIX} \) again. Down to \( 1:[A] \). \( \text{ENTER} \). Answer is displayed.

The computations for 3×3 determinants are messier than for 2×2’s. Various methods can be used, but the simplest method is probably the following:

- Write down the determinant
- Expand the determinant by rewriting the first two columns of numbers
- Then multiply along the down-to-the-right-diagonals
- and multiply along the down-to-the-left-diagonals
- Add the down-right-diagonals and subtract the down-right-diagonals
Example:

\[
A = \begin{pmatrix}
4 & -2 & 0 \\
-3 & 10 & 1 \\
2 & 6 & -1
\end{pmatrix}
\]

\[
\text{det } A = \begin{vmatrix}
4 & -2 & 0 \\
-3 & 10 & 1 \\
2 & 6 & -1
\end{vmatrix}
\]

Expand:

\[
4 \cdot (10) \cdot (-1) + (-2) \cdot (1) \cdot (2) + (0) \cdot (-3) \cdot (6) \
\]

\[-40 - 4 + 0 = -44
\]

\[-44 - 18 = 62
\]

A 3x3 determinant may be calculated on a calculator using the same steps as those for a 2x2

**UNDERSTAND THAT YOU MUST BE ABLE TO SOLVE A 3X3 DETERMINANT USING THE LONGHAND METHOD!**

**FIND THE DETERMINANT FOR THE FOLLOWING MATRICES:**

\[
\begin{pmatrix}
7 & 2 \\
0 & -3
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 2 & 5 \\
3 & 1 & 0 \\
1 & 2 & 1
\end{pmatrix}
\]